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Interaction between Trade and Environment Policies with Special Interest Politics: A Case when Commodity Markets are Imperfect

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Interaction between Trade and Environment Policies with Special Interest Politics: A Case when Commodity Markets are Imperfect

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Abstract

In the ambit of politics of special interest groups, this paper addresses the linkages between trade and environmental policies in imperfect commodity markets. A duopoly market is characterised by two-way trade in differentiated products that are polluting in nature. Firms in each country act as Cournot competitors in the international market and have direct stakes in both trade and environmental policies set by the incumbent government. Results suggest that in comparison to the baseline scenario where the incumbent is purely benevolent, the political equilibrium is characterised by higher import tariffs and lower pollution taxes. The voting support from political action groups (here firms) induces the incumbent to choose policies that favour them in general. However, the degree of product differentiation tends to bring down the divergence in policy outcomes under the two scenarios. Interdependencies between trade and environmental policies are also affected by the existence of producer lobbies. Unlike in case of the social optimum, the incumbent faces a trade-off between considerations for campaign funds from lobbies and the welfare motive for the population in general. Our study shows that environmental polices act as strategic substitutes, and trade policies may or may not be strategic substitutes. We find that complementarities in trade policies (a tariff war) arises only when incumbent governments are less corrupt. This outcome is attributed to the interdependencies between trade and environmental policies. For a less corrupt policy maker, the loss in campaign funds is outweighed by the net gains from environmental regulations, viz. improved environmental quality, gain/loss in consumer and producer surplus, pollution tax revenue, and tariff revenue. Therefore, the likelihood of trade wars would be lower if politicians are more corrupt.

JEL Classification: C7, D82, F18, P16.

Keywords: Special Interest Politics, Duopoly, Cournot Competition, Import Tariff, Pollution Tax, Product Differentiation.

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1 Introduction

The conflict between international trade and environmental policies is not a new economic phenomenon. There has been a proliferation in the number of international treaties and agreements as well as national laws pertaining to environmental protection. Furthermore, rapid expansion in international trade and trade agreements (plurilateral and multilateral) have exposed nations to the arduous task of dealing with the environmental impacts of trade and investment flows. While developed countries are in favour of linking trade and environmental policies, the less developed ones have been wary of losing to international competition from linked policy negotiations as more stringent environmental policies pose barriers to trade flows. There is evidence that developed countries have resorted to stricter environmental laws as substitutes to trade barriers when the World Trade Organisation (WTO) regulations of liberal trade policies are in place. The ban on tuna imports from Mexico by the US government in the year 1990 is an example of how developed nations use (abuse) surrogate policies such as environmental norms to restrict trade. Later in 2001, France imposed an import ban on asbestos and asbestos-containing products citing risks on human health. Recently, the European Union (EU) extended the Emissions Trading Scheme (ETS) on foreign airlines flying into the European airspace. This policy of 2012 received striking opposition from its trading partners like the US, China, India and Russia (Wu and Salzman, 2013). Hence, these measures (sometimes, ostensible) designed to ensure safety to human life have severe implications on trade flows. Nations which endorse and exercise stringent environmental safety standards limit market access to the exporters around the world, thereby jeopardising their trade competitiveness. The interesting aspect here is that such safety standards work as trade restriction measures (such as an import tariff/quota). As a result, nations committed to free trade or minimal trade restrictions have an incentive to use (or misuse) environmental regulations for domestic protection. This shows that there exist linkages between trade and environmental policies.

In recent years, there has been a dramatic shift in the discourse on green policies as far as developing nations are concerned. For instance, countries like India and China have made substantial progress in harnessing and deployment of cleaner energy. Such measures send a strong message to the international fora with regard to commitment and probity of nations to combat the pertinent issues of climate change and greenhouse gas (GHG) emissions (Mehra and Bhattacharya, 2019). As a result, prospects of international competition in achieving smoother transition to clean energy have expanded. Both developed and developing countries are embracing policies related to cleaner energy generation and deployment. As nations compete in green industrial policies in terms of subsidies, tax concessions and R & D expenditures, and engage in trade in equipments used in renewable energy generation, trade and environment conflicts become inevitable and more complex (Hajdukiewicz and Pera, 2020). While the energy transition driven by green industrial policies would have an impact on the environmental quality, it would have severe implications on trade flows as well as trade policies. The most compelling evidence in favour of the above argument are episodes of trade disputes between China, India and the US in the last decade. The dominance of China in the renewable energy market has challenged the domestic industry of its major trading partners, India and the US. In the year 2012, the US slapped countervailing duties and anti-dumping tariffs on solar panels in defence of its domestic manufacturers. The US has also dragged India to the dispute settlement body of the WTO against the local content requirement for solar cells in India's Jawaharlal Nehru National Solar Mission (JNNSM) Programme in 2013. As Hajdukiewicz and Pera (2020) point out that these contentious issues are also driven by domestic lobbying by solar manufacturers, growing protectionism and economic nationalism have also affected trade relations between trading partners who are signatories to a host of regional trade agreements (RTAs). For example, amidst growing border tensions between India and China [who are members of the Asia-Pacific Trade Agreement (APTA) with effect from January 1, 2002 (WTO, 2020)] since May 2020, trade relations between the Asian giants have soured. In August 2020, India raised tariffs on imports of green energy equipments from China (a 20 per cent basic customs duty on solar modules, cells and inverters) (Bhaskar, 2020; Stoker, 2020).

While the primary objective of the WTO is to push for phased reductions in trade barriers and promote "free and fair play", there also exists a complementary goal of sustainable development and environmental protection. Apparently, freer trade accompanied with environmental protection is considered to be welfare improving. However, very few countries have adopted these twin goals. International trade policies are inefficiently protective,[1] if not

[1]The US-China trade war in the year 2018 witnessed an import tariff on products like steel and aluminium by the US and further retaliation by China which slapped additional duties on American products including chemicals, vehicles, medical equipment, etc.

prohibitive, and environmental policies do not reflect true social costs of environmental damage. These policy distortions are aggravated when there exists market imperfections, monopoly pricing and asymmetric information. Therefore, all of these factors add yet another layer of distortion. Since the world economy is characterised by each of these attributes, thence emerges the pertinent question as to why are sub-optimal policies chosen in the first place? To answer this, we need to deviate from the paradigm of 'public interest', which is based on the assumption that policy makers are benevolent and maximise social welfare. Furthermore, the theory of 'public interest' qualifies to be a normative aspect when we analyse government intervention in policy-making. And, the positive aspect belies the primary motive of achieving efficiency through both 'first best' and 'second best' outcomes in dealing with economic problems. This is because policy-making processes are entrenched in the interplay of economics and politics.

Generally, incumbent governments seek to maximise political support to be in power. Similarly, there exists political action groups who have stakes in policies set by the regulator. As a result, these action groups make campaign contributions to the incumbent government to sway policies in their favour. According to Olson (1965), this phenomenon could be explained by the theory of collective action when a few economic agents succeed in overcoming the free rider problem and organise themselves into lobbies. Furthermore, per capita stakes vary inversely with the size of the group and hence a smaller group would have greater incentives to influence policy outcomes. Therefore, any change in economic policies would be guided by the preferences of interest groups. For example, the price of a polluting good of an industry or a firm sold in the domestic market or abroad would be affected by changes in both trade and environmental policies While the producer of an importable good, which is polluting in nature has an incentive to influence the government to lower environmental regulations and raise the level of import tariff, environmentalists would seek the opposite.[2] Consequently, both trade and environmental policies may be endogenously determined and the outcomes are unlikely to be the same as the ones that would be set by a benevolent social welfare maximiser. Further, a second tier of divergence in policy outcomes arises due to imperfections in the market. The dead weight loss associated with imperfect markets might lead to equilibrium policy outcomes which differ from those arising in case of perfectly competitive markets. There are possibilities of strategic interactions between producers within a nation as well as across nations which adds on to the intricacies. Lastly, the equilibrium policies would also depend on how: (a) national governments influence world prices through trade policy instruments like tariffs and subsidies (instances of interactions between large open economies) and (b) domestic producers exercise certain degrees of market power in the international market by competing in outputs or prices in an oligopolistic framework. Therefore, the political economy effects are likely to have distinct and contrasting implications thereof.

Since market imperfections are quite common in the world economy, it is imperative to probe the implications thereof. Therefore, this study addresses the aspect of the political economy of trade and environmental policies under imperfect competition in commodity markets.

The literature on international trade and imperfect competition can be classified under two broad categories, namely, trade in monopolistically competitive markets; and oligopoly and trade. The pioneering works by Krugman (1979) and Krugman (1980) focus on the aspect of increasing returns to scale in production and international trade based on monopolistic competition. It is evident that economies with identical tastes, similar technology and opportunity costs of production also trade with each other. Such a situation arises when firms incur high fixed costs of production, thereby restricting entry of a large number of potential producers in the market. Therefore, each of the existing firms, less in number, produce more to serve the market. It is due to high fixed costs of production that enable firms to spread it by producing more. As a result, average costs keep falling with expansion in output yielding increasing returns to scale. In addition, firms have an incentive to produce differentiated products in order to compete with their rivals. On the demand side, consumers prefer to consume a variety of goods and services which are close substitutes but no one variety is intrinsically superior to the other. Each variety is distinct and consumers may buy different varieties at the same price depending on their preferences. Their preferences are of

(CWS, 2018).

[2] A survey conducted by Saha (2015) for the period 2010-2014 finds that on an average 85 per cent of the manufacturing firms in India actively lobby the government. According to Kelemen and Vogel (2010), the EU has been emerging as a global advocate of environmental agreements on account of the emergence of green lobbies like the Brussels-based campaign group, the European Environmental Bureau (EEB), Kyoto Club, Greenpeace, etc. which push for more stringent regulatory stance by the policy makers.

the Dixit-Stiglitz type (Dixit and Stiglitz, 1977). International trade between such nations is welfare improving as firms can economise on costs of production and consumers can consume a greater variety of goods at lower prices. The entire premise of Krugman's theory of intra-industry trade thrives on the similarity hypothesis of Linder (1961), which emphasises on the role of home demand. Since knowledge is costly, entrepreneurs, in the beginning of a venture, would explore the internal demand. As market expands, firms are more likely to look for profit opportunities beyond local markets. At this juncture, firms enter the export market and serve only those foreign markets where the demand is similar to the local market. In short, existence of home demand is a necessary condition for monopolistically competitive firms to enter the export market.

On the other hand, the aspect of intra-industry trade in oligopoly markets was studied by Brander (1981), Brander and Krugman (1983), Brander and Spencer (1985), Dixit (1984), Dixit (1988) and Eaton and Grossman (1986). These studies also focus on horizontal product differentiation. While Brander (1981) and Brander and Krugman (1983) emphasised on intra-industry trade in both identical and differentiated products by exploring possibilities of cross-hauling and reciprocal dumping, the others studied the welfare impacts of both trade policies (export/import taxes and subsidies) and industrial policies (production tax and subsidies). The primary conclusion of these studies is that interventionist trade policies may improve welfare if markets are imperfectly competitive. When domestic and foreign firms compete in output in order to serve a third-country market, the Cournot-Nash equilibrium level of trade policy is an export subsidy (Brander and Spencer, 1985; Dixit, 1984). In a similar fashion, Eaton and Grossman (1986) studied welfare impacts of trade and industrial policies in an oligopoly-setting by assuming arbitrary conjectural variations. The findings suggest that domestic policies are unambiguously welfare improving since they narrow the divergence between prices and marginal costs.

There have been studies on vertical product differentiation and trade as well (Motta, 1994; Zhou et al., 2002). Motta (1994) argues that internationalisation decisions of firms depend on the size of the domestic market. Firms operating in the large country export better quality product in comparison to its rivals in the small country. As the quality gap rises, firms in the small country not only lose to foreign competition but also face difficulty in serving the domestic market. Zhou et al. (2002) extend Motta's argument by examining equilibrium domestic policies (investment tax/subsidy) when firms in a developed country compete with firms in a less developed country (LDC) to serve third-country export markets. Equilibrium policies vary depending upon the type of duopoly competition, viz., Cournot competition and Bertrand competition as well as on the relative position of firms on the quality ladder.

Further contribution to the literature on trade and imperfect markets is attributed to the studies which juxtapose the implications of environmental regulations and their linkages with trade policies (Barrett, 1994; Burguet and Sempere, 2003; Conrad, 1993, 2001; Jinji, 2005; Neary and Leahy, 2000; Tanguay, 2001; Ulph and Ulph, 1996; Walz and Wellisch, 1997). These studies analyse environment and trade policies in an oligopolistic framework. Most of these studies examine the welfare implications of trade and environmental policies. A broader conclusion of these studies is that unlike weaker environmental regulations, export subsidies are welfare improving. Weaker environmental regulations are typically costly to the society as they worsen the environmental quality. Export/production subsidies, on the other hand, are not costly to the society since they redistribute resources from the government to industries/firms. Barrett (1994) argues that production/export subsidies work better than weaker environmental standards when it comes to improving international competitiveness of domestic firms. Both the studies by Walz and Wellisch (1997) and Jinji (2005) suggest that environmental costs due to local pollution can be fully internalised through emission tax when countries can choose direct export subsidies. Apart from emission taxes as potential policy instruments to control for externalities from pollution, subsidies on pollution abatement and investment credits on cleaner technologies may improve international competitiveness of exporting firms (Conrad, 2001; Neary and Leahy, 2000). While most of these studies rationalise restricted trade policies in an oligopoly framework, Burguet and Sempere (2003) argue that trade liberalisation would improve welfare if environmental taxes exist, since national governments have an incentive to use environmental policy as a substitute to restrict trade. The result stands in sharp contrast to the earlier studies on account of the assumption of bilateral trade policies. In a Brander-Spencer setting of bilateral trade with reciprocal dumping, the authors show that a bilateral tariff reduction would expand output of firms operating in each of the trading nations, assuming that tariffs are one of the components of cost faced by the firms. The welfare implications would then, typically depend on whether prices exceed (fall short of) the marginal social cost associated with local environmental damage from production of the trade good. Welfare would improve (fall) if price of the good exceeds (is below) the marginal social cost of pollution.

However, these studies do not incorporate special interest politics in their analyses. In the ambit of strategic trade policy, the inherent possibilities for rent transfer from trading partners may be followed by rent-seeking activities by domestic beneficiaries including firms, industries, or political agents. Lee (2003) explores this aspect by re-examining the Brander and Spencer (1985) formulation in the presence of domestic lobbies. The politically determined equilibrium tariff in the domestic country is higher than the one chosen by a benevolent dictator. This result closely resembles the case of a unilateral export-subsidy scheme introduced by the home country (Kagitani, 2008). It is notable that these studies do not incorporate the aspect of product differentiation and trade as in Fung et al. (2009) and Kagitani (2009). Notwithstanding the fact that studies on lobbying in strategic trade policies explore both Cournot and Bertrand competition with and without product differentiation, the implications of two-way trade is quite unexplored. One can very well expect two-way trade between nations as opposed to the standard formulation of duopolies competing in third country markets. It is one of the pioneering studies by Kim (2017) that lays emphasis on the role of political action groups in the determination of tariffs in case of intra-industry trade. Also, unlike the earlier studies on the political economy of trade policy, this study makes a novel attempt to show that import competing firms may not have political objections to trade liberalisation when products are highly differentiated and foreign markets follow the norm of reciprocity. Besides, consumers' love for a variety of products precludes possibilities of losing domestic market share to foreign imports. Furthermore, productive firms which enter the export market are more likely to lobby for trade liberalisation, and more so when their products are highly differentiated. This result is consistent with the adoption of bilateral trade liberalisation that fosters greater market access. Therefore, product differentiation revamps the entire political dynamics.

In the present study, we develop a political economy model of intra-industry trade between two countries, each with one firm, where governments can avail both tariffs and pollution taxes as potential policy instruments. Until recently, Shao et al. (2018) have examined the effects of international trade on regulations on conservation of exhaustible resources in a Cournot setting, where governments are industry-biased. Firms in each country engage in reciprocal dumping to capture profits in the world market. Results suggest that the resource tax on harvest of the exhaustible resource rises as countries open up for trade compared to the corresponding tax under autarky. The higher resource tax as opposed to the tax under autarky leads to a reduction in resource extraction and output and increases the level of resource conservation. The primary argument related to the above result is that an industry-biased government is less likely to care about environment quality and resource conservation under autarky. As countries open up for trade, international competition between home and foreign firms as well as the governments induces a race to the top with respect to harvest taxes. Therefore, the study revolves around harvest taxes and does not incorporate trade policies. Our model, on the contrary, explicitly studies trade policy along with environmental policy and the associated interactions between the two. We also consider trade in a differentiated product which is associated with a negative production externality. Furthermore, we carry out a focused analysis of interactions between the domestic and the foreign markets through both tariffs and pollution taxes. There is a resultant shift in the policy space from a uni-dimensional (as in Kim (2017)) to a multi-dimensional one along with the likelihood of pairwise interdependencies in policies.

The paper has been organised as follows. We formulate a model of two-way trade in horizontally differentiated products between two large open economies, each with one firm. The firm in the home country, which produces a differentiated variety of the traded good, act as a Cournot competitor with the foreign firm. Production of the traded good generates local pollution and the associated environmental damage enters the consumer's utility function as disutility from production. The government regulates pollution by imposing a tax on production. The existence of imperfect competition creates rent seeking incentives for national governments who try to appropriate profits from the international market through policies regulating trade flows. All proceeds from regulation are redistributed in a lumpsum fashion to the entire population so as to avoid any more distortions in the economy. Clearly, regulation of trade flows and environmental pollution also affect the price of the traded good, thereby, inducing firms to lobby on both trade and environmental policies. We begin by characterising the social optimum whose outcomes form the benchmark, followed by solving the political equilibrium.

The three-stage game is solved through backward induction. In the first stage, the firm in each country non-cooperatively chooses its campaign contribution schedule subject to a set of policy outcomes that maximise its welfare. The policy basket comprises an import tariff and a pollution tax. In the second stage, the policy maker/incumbent politician sets policies and receives campaign funds from the firm. This is followed by the third stage where labour and world markets clear and production and consumption take place.

Results suggest that the non-cooperative equilibrium tariff and pollution tax differ across the benchmark scenario and the political equilibrium. Equilibrium import tariffs are higher than those obtained under the social optimum. The political inclination induces the government to set a pollution tax that is lower than the socially optimal level. These results are are consistent with those obtained by Grossman and Helpman (1995), Mehra (2010) and Damania et al. (2003). We observe that these divergences are reduced when the policy maker becomes less corrupt, i.e., his/her preference towards aggregate social welfare increases. Also, a rise in the degree of product differentiation works as a safety-net for the Cournot firm and reduces its incentive to actively lobby for domestic protection. This is because consumers consume both the domestic as well as the foreign variety and a rise in the degree of differentiation between the two cuts down the threat of losing the domestic market share amidst the presence of the foreign variety.

We characterise trade and environmental policy interdependencies under the social optimum and the political equilibrium by considering the impact of a change in each of these policies on the marginal effects of these policies on the following components of the objective function of the policy maker: (a) consumer surplus; (b) firm's profits; (c) pollution tax revenue; (d) environmental damage; (e) tariff revenue; and (f) campaign contributions (an additional component when the policy maker has politically motivated). Under the social optimum, our findings reveal that both home (foreign) import tariff and pollution tax respond positively to a rise in home (foreign) pollution tax and import tariff. As environmental regulations tighten in the home (foreign) country, the domestic (foreign) firm becomes less competitive due to a rise in cost and this increases the rent-capturing incentive of the home (foreign) government which is reflected in higher trade protection. When import tariffs in the home (foreign) country rises, the home (foreign) firm becomes more competitive and the home (foreign) government's incentive to capture rents from its rival falls. This results in an increase pollution tax in the home (foreign) country. The effect of a rise in foreign (home) import tariff also leads to a rise in home (foreign) pollution tax. If the foreign (home) country raises protection, the negative effect of increased pollution tax on the home (foreign) firm's profits falls. On account of Cournot competition, a rise in trade protection in the foreign (home) country leads to a reduction in output of the home (foreign) firm's production for exports in the other country. This mitigates the effect of decreased marginal revenue from sales in the foreign (home) country as well as the cost disadvantage from increased tax on pollution. Therefore, home (foreign) pollution tax rises. However, home (foreign) import tariff responds inversely to an increase in foreign (home) pollution tax. Again, Cournot competition results in increased competitiveness of the home (foreign) firm in response to a rise in pollution tax in the foreign (home) country, thereby tempering the rent-capturing incentive of the home (foreign) government which translates into lower trade protection. Also, environmental policies at home and foreign are inversely related. As environmental regulations tighten in the foreign (home) country, home (foreign) output expands and this subsequently aggravates the negative effect of a rise in home (foreign) pollution tax on the home (foreign) firm's profit. This leads to a reduction in home (foreign) pollution tax. Home (foreign) pollution tax also moves downward (upward) when pollution tax revenue is increasing (decreasing) in the tax rate, i.e., output is less (highly) responsive to changes in the tax rate. The net effect of the two factors which pulls the tax downward, outweighs the effect which pulls it in the upward direction. This is due to the marginal gain in consumer surplus from an increase in foreign (home) pollution tax which offsets the price rise of the output in the home (foreign) country. Hence, the home (foreign) government sets a less stringent environmental policy. Finally, home (foreign) import tariff rises in response to a rise in foreign (home) import tariff. It is notable that this emanates from a second-order effect, since market segmentation generates a zero first-order effect. A rise in trade protection in the foreign (home) country raises the pollution tax in the home (foreign) country which further raises the level of trade protection in the home (foreign) country.

In comparison to the benchmark, policy interdependencies in case of the political equilibrium are affected by the government's considerations for campaign funds. The direction of movement of home (foreign) tariff in response to a rise in both home (foreign) pollution tax and foreign (home) pollution tax is ambiguous. When the politically motivated government is highly corrupt, the direction of movement reverses in comparison to that of the social optimum. Similarly, the response to a rise in home (foreign) import tariff on home (foreign) pollution tax is less predictable. Such ambiguities arise due to the relative strengths of the two effects, namely, considerations for aggregate social welfare of the general population; and considerations for campaign funds which directly impact the likelihood of being re-elected for the next term of political office. While the political consideration tends to lower home (foreign) pollution tax, considerations for aggregate social welfare leads to the tightening of environmental regulations at home (foreign).[3] The reverse holds true when we consider the relationship between foreign (home) pollution tax and home (foreign) tariff. Finally, home (foreign) pollution tax increases when: the foreign (home) country raises its import tariff; the foreign (home) country reduces pollution tax. The policy maker's social welfare and political considerations tend to work in the same direction. Lastly, owing to the ambiguity in interdependencies between home (foreign) pollution tax on home (foreign) tariff, the direction of movement of tariffs is indeterminate. We also undertake a numerical simulation exercise along with a set of regulatory conditions based on some parametric restrictions which confirms these results.

The following section (Section 2) illustrates the analytical model. Section 3 concludes.

2 The Analytical Framework

We consider a symmetric international duopoly model with one firm located in each of the two countries, namely, home and foreign, based on the framework developed by Dixit (1988). Each firm produces a unique variety of the differentiated good. The assumption of two varieties of the product being produced ensures mathematical tractability. All notations related to the home country are appended with a $(*)$ in order to denote the foreign country counterparts.

2.1 Consumers

There are n consumers at home, each with identical preferences. Each consumer maximises the following quasi linear utility function

$$U = z + u(x, y) - \theta(X + X^*); \tag{1}$$

where z denotes the consumption of the numeraire good Z whose price in the domestic and the foreign market is unity and x and y refer to demand for outputs X and Y respectively. The marginal damage from production at home, θ is assumed to be exogenous. Let $\widetilde{X}$ $(= X + X^*)$ denote total production of the home variety, where X refers to production for home consumption and X^* refers to exports to the foreign country. The function $u(.)$ is differentiable, increasing and concave in each of its arguments. Pollution is a local bad[4] and enters the utility function as negative utility.

The budget constraint faced by an individual at home is given as

$$px + qy + z = m; \tag{2}$$

where p, q denote domestic prices of X and Y respectively and m denotes income.

Utility maximisation yields inverse demand functions as

$$p = p(x, y); \tag{3}$$

with $p_x < 0$ and $p_y < 0$.

$$q = q(x, y); \tag{4}$$

with $q_x < 0$, $q_y < 0$.

Further, we also assume that own-price effect is larger than cross-price effect, i.e.,

$$|p_x| > |p_y|; \ |q_x| < |q_y|. \tag{5}$$

[3]This ties in with the results obtained by Damania et al. (2003) and Fredriksson and Muthukumara (2004) in case of trade in small open economies with the assumption of perfectly competitive markets and Straume (2006) for commodity trade in duopoly markets. This is in sharp contrast to the result in the study by Mehra (2010) where the policy maker contains the welfare loss of higher import tariffs through the imposition of stricter environmental taxes for the protected sector. Unlike our study, Mehra (2010) employs a Nash bargaining game between the incumbent and the lobby.

[4]A local bad refers to a good whose production and/or consumption generates negative externality (environmental damage) that is confined to the national boundary of the country where production and/or consumption takes place.

This helps in ensuring uniqueness of equilibria.

Demand functions are concave in prices, i.e.,

$$p_{xx} \leq 0;\ p_{yy} \leq 0;\ q_{xx} \leq 0;\ q_{yy} \leq 0. \tag{6}$$

In order to ensure mathematical tractability, demand functions are also assumed to be symmetric across nations such that own price effects and cross price effects are identical across varieties, i.e.,

$$p_x = q_y;\ p_y = q_x;\ p^*_{x^*} = q^*_{y^*};\ p^*_{y^*} = q^*_{x^*}. \tag{7}$$

Consumer surplus is given by

$$S(p,q) = u(x,y) - p(x,y)x - q(x,y)y. \tag{8}$$

This completes the consumption side.

2.2 Producers

The numeraire good Z is produced using homogeneous labour input L under constant returns to scale in a perfectly competitive market setting. By appropriate choice of units, the labour-output coefficient is assumed to be unity, i.e., one unit of labour produces one unit of output Z, with wage rate equal to unity. The aggregate labour supply is sufficiently large to ensure a positive level of output of Z. There is no pollution associated with production or consumption of Z. The differentiated good, $\widetilde{X}$, is produced with constant marginal cost c. Pollution damages which arise from the production of X and X^* are subject to a charge imposed by the policy maker, say a pollution tax. Furthermore, the foreign country imposes a tariff on the exports by the domestic firm.

Profits of the home firm are given as

$$\pi = p(x,y)X + p^*(x^*,y^*)X^* - c(X+X^*) - \tau(X+X^*) - t^*X^*. \tag{9}$$

The first two terms on the right-hand side of equation (9) refer to the total revenue from sales in the domestic market and the foreign market respectively. The third term is the linear cost function. τ and t^* denote the pollution tax rate imposed by the home government and the tariff on imports of X^* imposed by the government in the foreign country respectively.

Analogously, profits earned by the foreign firm are given as

$$\pi^* = q(x,y)Y + q^*(x^*,y^*)Y^* - c^*(Y+Y^*) - \tau^*(Y+Y^*) - tY; \tag{10}$$

where τ^* and t denote pollution tax rate imposed by the foreign government and import tariff faced by the foreign firm respectively.

It is important to note here that the markets are segmented, i.e., each firm considers each country as a separate market and makes distinct output decisions for each. The duopoly firms act as rivals in the world market and try to extract profits by capturing a greater market share. Market segmentation also implies that any policy distortion that is introduced to regulate foreign output meant for sales within the domestic territory (say, an import tariff) has no impact on the foreign output sold in the foreign market. This gives each country's governments greater flexibility to address distortions in owing to market imperfections. On the contrary, environmental policies like pollution tax are meant for regulating domestic output by internalising the negative externality. Therefore, home pollution tax affects domestic production meant for home consumption as well as exports to the foreign country. Similarly, foreign pollution tax affects the output of the foreign firm which is consumed within its territories as well as exports to the home country.

The market clearing condition requires demand to be equal to supply. With identical preferences of consumers, we have

$$nx = X; n^*x^* = X^*; ny = Y; n^*y^* = Y^*; nz = Z; n^*z^* = Z^*. \tag{11}$$

2.2.1 Firm's problem and equilibrium conditions

The domestic firm faces the following problem:

$$\max_{(X,X^*)} \pi(X, X^*)$$

The first-order conditions for profit maximisation are

$$\frac{\partial \pi}{\partial X} \equiv \pi_X = 0; \tag{12}$$

i. e.,

$$\frac{X}{n} p_X + p = c + \tau; \tag{13}$$

and

$$\frac{\partial \pi}{\partial X^*} \equiv \pi_X^* = 0; \tag{14}$$

i. e.,

$$\frac{X^*}{n^*} p^*_{X^*} + p^* = c + \tau + t^*. \tag{15}$$

These give us the best response functions of the home firm for the home variety in home and foreign markets respectively as

$$X = X(p, q, \tau; n, c); \tag{16}$$

$$X^* = X^*(p^*, q^*, \tau, t^*; n^*, c). \tag{17}$$

Similarly, the foreign firm faces the following problem:

$$\max_{(Y,Y^*)} \pi^*(Y, Y^*)$$

The first-order conditions for profit maximisation are

$$\frac{\partial \pi^*}{\partial Y} \equiv \pi^*_Y = 0; \tag{18}$$

i. e.,

$$\frac{Y}{n} q_Y + q = c^* + \tau^* + t; \tag{19}$$

and

$$\frac{\partial \pi^*}{\partial Y^*} \equiv \pi^*{}_{Y^*} = 0; \tag{20}$$

i. e.,

$$\frac{Y^*}{n^*} q^*_{Y^*} + q^* = c^* + \tau^*. \tag{21}$$

These give us the best response functions of the foreign firm for the foreign variety as

$$Y = Y(p, q, \tau^*, t; n, c^*); \tag{22}$$

$$Y^* = Y^*(p^*, q^*, \tau^*; n^*, c^*). \tag{23}$$

Each of these first-order conditions show that marginal revenue from each market is equated to marginal costs.

While equations (16) and (22) determine the equilibrium output for home country consumption, equations (17) and (23) determine the equilibrium output for foreign country consumption.

Now, the solutions given by the expressions (16), (17), (22) and (23) are optimal only if the following second-order conditions are satisfied, i.e.,

$$\pi_{XX} < 0;\ \pi_{X^*X^*} < 0;\ \pi^*_{YY} < 0;\ \pi^*_{Y^*Y^*} < 0. \tag{24}$$

We also impose the following restrictions

$$\pi_{XY} < 0;\ \pi_{X^*Y^*} < 0;\ \pi^*_{YX} < 0;\ \pi^*_{Y^*X^*} < 0; \tag{25}$$

and

$$\pi_{XX} < \pi_{XY};\ \pi_{X^*X^*} < \pi_{X^*Y^*};\ \pi^*_{YY} < \pi^*_{YX};\ \pi^*_{Y^*Y^*} < \pi^*_{Y^*X^*}. \tag{26}$$

Condition (25) states that own marginal revenue is declining in the level of output of the rival firm, which implies that the reaction curves are downward sloping. It turns out that outputs of the Cournot firms are strategic substitutes. Downward sloping reaction functions ensure stability of equilibria and if condition (25) holds globally, it implies uniqueness of equilibria. However, the negative sign is contingent upon the nature of the demand function. In case of concave demand functions, the mixed partials of the profit function would be negative (Brander and Krugman, 1983). Condition (26) implies that own effects of output on marginal revenue exceed cross effects, which ensures that the sufficiency conditions for local maximum are met.

Conditions (24)-(26) ensure existence, uniqueness and stability of equilibria with

$$D = \pi_{XX}\pi^*_{YY} - \pi_{XY}\pi^*_{YX} > 0; \tag{27}$$

$$D^* = \pi_{X^*X^*}\pi^*_{Y^*Y^*} - \pi_{X^*Y^*}\pi^*_{Y^*X^*} > 0; \tag{28}$$

which imply that the reaction curves cross each other once and only once, i.e., the best response functions satisfy the single crossing property. Finally, conditions (24)-(28) ensure sufficiency for local maxima.

In case the demand curves are linear, the second-order partial derivatives of the demand functions would be zero. As a result, the first term on the right-hand side of conditions (A1)-(A8) disappears [see Appendix A]. We now employ the assumption of symmetric demand functions across nations (given by expression (7)) which further simplifies the set of conditions (24)-(26) as

$$\begin{aligned} \pi_{XX} = \frac{2}{n}p_X;\ \pi_{XY} = \frac{1}{n}p_Y;\ \pi_{X^*X^*} = \frac{2}{n^*}p^*_{X^*};\ \pi_{X^*Y^*} = \frac{1}{n^*}p^*_{Y^*}; \\ \pi^*_{YY} = \frac{2}{n}q_Y = \frac{2}{n}p_X;\ \pi^*_{YX} = \frac{1}{n}q_X = \frac{1}{n}p_Y; \\ \pi^*_{Y^*Y^*} = \frac{2}{n^*}q^*_{Y^*} = \frac{2}{n^*}p^*_{X^*};\ \pi^*_{Y^*X^*} = \frac{1}{n^*}q^*_{X^*} = \frac{1}{n^*}p^*_{Y^*}. \end{aligned} \tag{29}$$

The set of conditions in (29) can be substituted in (27) and (28) to get the final expressions as

$$D = \frac{1}{n^2}\left(4p_X^2 - p_Y^2\right) > 0; \tag{30}$$

$$D^* = \frac{1}{n^{*2}}\left(4{p^*}^2_{X^*} - {p^*}^2_{Y^*}\right) > 0. \tag{31}$$

Since own price effects exceed cross price effects, the terms in parenthesis on the right-hand side of expressions (30)and (31) would be positive. In other words, the sign of the determinant of the Hessian matrix for profit maximisation would be positive [see Appendix A and Appendix B].

2.3 Government

The government in each country chooses a tariff and a pollution tax that maximise its objective function. In case the government is benevolent, it maximises the aggregate social welfare which is a sum of consumer surplus, producer surplus, tariff revenue and pollution tax revenue net of environmental damage. In contrast, the government with political objectives (seeking political office in the next term) tries to maximise the likelihood of being re-elected and attaches some weight to campaign contributions made by political action groups. Consequently, the objective function comprises both aggregate social welfare and political contributions.

We now turn to provide the comparative static results associated with the effects of changes in pollution taxes and tariffs on output, which are used later in the analysis to explain some results. These pertain to firm's optimisation and will hold irrespective of whether the government is benevolent or politically motivated.

2.3.1 Comparative static effects of tariffs and pollution taxes

We compute the comparative static effects of pollution taxes and tariffs by totally differentiating the first-order conditions (12), (14), (18) and (20) which yields that,

$$X_t \equiv \frac{dX}{dt} = -\frac{\pi_{XY}}{D} > 0;\ X^*_t \equiv \frac{dX^*}{dt} = 0; \tag{32}$$

$$X_{t^*} \equiv \frac{dX}{dt^*} = 0; \; X^*_{t^*} \equiv \frac{dX^*}{dt^*} = \frac{\pi^*_{Y^*Y^*}}{D^*} < 0; \tag{33}$$

$$Y_t \equiv \frac{dY}{dt} = \frac{\pi_{XX}}{D} < 0; \; Y^*_t \equiv \frac{dY^*}{dt} = 0; \tag{34}$$

$$Y_{t^*} \equiv \frac{dY}{dt^*} = 0; \; Y^*_{t^*} \equiv \frac{dY^*}{dt^*} = -\frac{\pi^*_{Y^*X^*}}{D^*} > 0; \tag{35}$$

$$X_\tau \equiv \frac{dX}{d\tau} = \frac{\pi^*_{YY}}{D} < 0; \; X^*_\tau \equiv \frac{dX^*}{d\tau} = \frac{\pi^*_{Y^*Y^*}}{D^*} < 0; \tag{36}$$

$$X_{\tau^*} \equiv \frac{dX}{d\tau^*} = -\frac{\pi_{XY}}{D} > 0; \; X^*_{\tau^*} \equiv \frac{dX^*}{d\tau^*} = -\frac{\pi_{X^*Y^*}}{D^*} > 0; \tag{37}$$

$$Y_\tau \equiv \frac{dY}{d\tau} = -\frac{\pi^*_{YX}}{D} > 0; \; Y^*_\tau \equiv \frac{dY^*}{d\tau} = -\frac{\pi^*_{Y^*X^*}}{D^*} > 0; \tag{38}$$

$$Y_{\tau^*} \equiv \frac{dY}{d\tau^*} = \frac{\pi_{XX}}{D} < 0; \; Y^*_{\tau^*} \equiv \frac{dY^*}{d\tau^*} = \frac{\pi_{X^*X^*}}{D^*} < 0. \tag{39}$$

Expressions (32)-(35) show that home market trade policies affect only the home market. This is driven by the assumption of constant marginal costs. This separation result would be lost if marginal costs depended on output levels (Dixit, 1984). A rise in tariffs at home raises the production of the import competing home variety and reduces imports. Furthermore, domestic production of the polluting good is decreasing in home pollution taxes designed to regulate the negative production externality. Owing to the fact that the firms are Cournot players, a rise in pollution taxes in one country raises the output of the firm in the other country (shown by (36)-(39)) [see detailed proof in Appendix B].

Proposition 1: *An increase in home (foreign) import tariff :*

(i) leads to a rise in output meant for sale in the local market;

(ii) has no effect on the output exported.

Proposition 2: *An increase in home (foreign) pollution tax leads to a fall in domestic (foreign) output and a rise in foreign (domestic) output.*

To be precise, as two open economies, home and foreign, each with only one Cournot firm producing a differentiated good which is polluting in nature engage in two-way trade, policy makers in each country undertake measures to control trade flows and environmental damages emanating from production of the polluting good. While the Cournot players compete in output, the policy maker's objective in each country hinges upon extraction of rents by raising the competitiveness of the firm operating in the country in question, either by raising domestic protection or through relaxations on environmental compliance norms, thereby generating a cost advantage for the firm and aggravating a cost disadvantage for its rival. This forms the essence of strategic trade and environmental policy. In the present scenario, market segmentation ensures that home (foreign) tariffs only affect the production meant for home (foreign) consumption and not the output meant for exports. Unlike tariffs, home (foreign) pollution taxes affect total production in the home (foreign) country, including the output that is exported. Other things remaining constant, a rise in home (foreign) pollution tax would raise the cost of production of the home (foreign) firm and thereby lead to a contraction in home (foreign) output. As Cournot competitors, any rise (fall) in the level of output of one of the players would lead to a reduction (increase) in the marginal profits of its rival. This implies that outputs of each firm are strategic substitutes.

Next, we outline the stages of the game.

2.4 Stages of the game

We solve for the sub-game perfect Nash equilibrium of the game which takes place in three stages. In the first stage, the Cournot firm in each country chooses a contribution schedule or bribes for the incumbent government contingent upon an array of policy outcomes that maximises its welfare. The firm in each country acts simultaneously and non-cooperatively, taking the contribution schedules of the other as given. Faced with the contribution schedule, the incumbent government or politician sets policies in the second stage. Observing the set of policies announced, the firm makes contribution to the government. In the final stage, labour and world markets clear and production and consumption take place.

In view of facilitating a comprehensive exposition of the political economy effects of trade and environmental policies in a duopoly framework, we solve for the social optimum (a baseline model) which is characterised by the second and the third stage of the game. In this model, political actions groups are either absent or the policy maker is benevolent and assigns a negligible or no weight to campaign contributions.

In the subsequent sections, we solve for the social optimum, which constitutes our baseline case, followed by the political equilibrium.

2.5 The social optimum

A benevolent social planner in each country sets pollution taxes and import tariffs by maximising aggregate social welfare. Aggregate social welfare is given by the sum of labour income, consumer surplus, producers' surplus, pollution tax revenues net of environmental damage and tariff revenue. Mathematically, the expression for aggregate social welfare is given as

$$W^G = l + nS(.) + \pi(.) + (\tau - n\theta)\widetilde{X} + tY; \tag{40}$$

where W^G denotes aggregate social welfare of the home country. The first-order conditions with respect to t and τ are

$$\frac{\partial W^G}{\partial t} = \frac{\partial (nS)}{\partial t} + \frac{\partial \pi}{\partial t} + \frac{\partial (\tau \widetilde{X})}{\partial t} - \frac{\partial (n\theta \widetilde{X})}{\partial t} + \frac{\partial (tY)}{\partial t} = 0; \tag{41}$$

$$\frac{\partial W^G}{\partial \tau} = \frac{\partial (nS)}{\partial \tau} + \frac{\partial \pi}{\partial \tau} + \frac{\partial (\tau \widetilde{X})}{\partial \tau} - \frac{\partial (n\theta \widetilde{X})}{\partial \tau} + \frac{\partial (tY)}{\partial \tau} = 0. \tag{42}$$

The policy maker at home takes foreign policies (t^*, τ^*) as given and chooses (t, τ) that maximise social welfare. Symmetric expressions can be obtained for the foreign country as well.

Both the first-order conditions (41)-(42) for the home country (and, similarly, for the foreign country), can be decomposed into five major components. These are effects of change of t (t^*) or τ (τ^*) on (a) consumer surplus; (b) profits; (c) pollution tax revenue; (d) environmental damage and (e) tariff revenue. The government in each country has to balance the above considerations while determining optimal tariffs and pollution taxes. These effects are purely partial equilibrium and market driven effects which may not be true in equilibrium and do not incorporate any political economy considerations. Furthermore, the effects derived are based on the assumption of linear demand functions. We focus on the home country and discuss each of these decomposed effects in some detail below. A subscript of t and τ to a variable denotes the partial effect of a change of the former on the latter.

2.5.1 Effect on consumer surplus

The effects of changes of t and τ on consumer surplus are obtained to be

$$\frac{\partial S}{\partial t} = \frac{1}{n}\Big(u_X X_t + u_Y Y_t - pX_t - Xp_t - qY_t - Yq_t\Big); \tag{43}$$

$$\frac{\partial S}{\partial \tau} = \frac{1}{n}\Big(u_X X_\tau + u_Y Y_\tau - pX_\tau - Xp_\tau - qY_\tau - Yq_\tau\Big). \tag{44}$$

Equations (43) and (44) show how the import tariff and the pollution tax alter domestic prices and demand and hence consumer surplus, respectively. Using the envelope property in the first-order conditions for utility maximisation simplifies equations (43) and (44) to

$$\frac{\partial S}{\partial t} = -\frac{1}{n}\Big(Xp_t + Yq_t\Big); \tag{45}$$

$$\frac{\partial S}{\partial \tau} = -\frac{1}{n}\Big(Xp_\tau + Yq_\tau\Big); \tag{46}$$

where

$$p_t = \frac{1}{n}\Big(p_X X_t + p_Y Y_t\Big) > 0; \tag{47}$$

$$q_t = \frac{1}{n}\Big(q_X X_t + q_Y Y_t\Big) > 0; \tag{48}$$

$$p_\tau = \frac{1}{n}\Big(p_X X_\tau + p_Y Y_\tau\Big) > 0; \tag{49}$$

and

$$q_\tau = \frac{1}{n}\Big(q_X X_\tau + q_Y Y_\tau\Big) > 0; \tag{50}$$

Expressions (47)-(50) refer to the effect of change of policy variables on domestic prices. These effects are obtained by differentiating the demand functions given in equations (3)-(4) partially with respect to t and τ. Equations (47)-(50) show that domestic prices are increasing in both import tariff and pollution tax. Since the inverse demand function of each variety of the traded good depends on the level of consumption of both the varieties, any change in domestic policy, be it an import tariff or a pollution tax, would comprise the partial effect of a change in quantity demanded of both varieties. The first term in parenthesis on the right-hand side of equations (47)-(50) refers to the policy induced change in domestic demand for the home variety followed by the second term which captures the resultant change in domestic demand for the imported variety. A tariff on the imports of the foreign variety increases its price at home (assuming that condition in (5) holds). This induces consumers at home to demand more of the domestic variety, thereby, raising its price and output at home. In contrast, a pollution tax on the production of the home variety leads to a rise in its price and fall in the domestic output, owing to a rise in the cost of production. This aggravates the under-provision problem associated with imperfect competition. Now, domestic demand shifts in favour of the foreign variety, thereby, raising its home price. Therefore, considerations for consumer surplus requires lower domestic protection as well as a less stringent environmental policy. These results are based on the assumption that own-price effects outweigh cross-price effects [see assumption in (5)].

Proposition 3 : *An increase in home (foreign) import tariff and pollution tax leads to a rise in domestic (foreign) price of both the home and the foreign variety of the traded good.*

Further, using the results in (47) and (48), and (49) and (50) in (45) and (46), respectively, we get

$$\frac{\partial(nS)}{\partial t} < 0; \tag{51}$$

$$\frac{\partial(nS)}{\partial \tau} < 0. \tag{52}$$

Similar results hold for the foreign country as well.

Hence, a rise in tariff or pollution tax in any country (home or foreign) results in a fall in the consumer surplus of the respective country due to a rise in domestic prices of both home and foreign varieties. From Proposition 3, we see that both import tariff and pollution tax lead to a rise in the domestic price of the traded good. This causes the consumer surplus to fall. We, therefore, have the following proposition:

Proposition 4: *An increase in home (foreign) import tariff and pollution tax leads to a reduction in domestic (foreign) consumer surplus.*

2.5.2 Effect on firm's profit

The profit of the firm at home depends on the level of its own output as well as the output level of its rival and the policies chosen by the social planner at home and in the foreign country. Hence,

$$\pi = \pi(X, Y, X^*, Y^*, t, \tau, t^*, \tau^*). \tag{53}$$

The effects of changes in t and τ at home are given as

$$\frac{\partial \pi}{\partial t} = \pi_X X_t + \pi_Y Y_t + \pi_{X^*} X_t^* + \pi_{Y^*} Y_t^* + \pi_t; \tag{54}$$

$$\frac{\partial \pi}{\partial \tau} = \pi_X X_\tau + \pi_Y Y_\tau + \pi_{X^*} X_\tau^* + \pi_{Y^*} Y_\tau^* + \pi_\tau. \tag{55}$$

From the first-order conditions of profit maximisation, we get $\pi_X = 0$ and $\pi_{X^*} = 0$. Hence,

$$\frac{\partial \pi}{\partial t} = \pi_Y Y_t + \pi_{Y^*} Y_t^* + \pi_t > 0; \tag{56}$$

$$\frac{\partial \pi}{\partial \tau} = \pi_Y Y_\tau + \pi_{Y^*} Y_\tau^* + \pi_\tau < 0. \tag{57}$$

In (56) and (57), the first two terms on the right-hand side give us the second-order effects followed by the first-order effect. In case of import tariff, market segmentation allows the domestic firm's profit to be unaffected directly by an import tariff. Therefore, the first-order effect turns out to be zero. As far as the second-order effects are concerned, the second term on the right-hand side of (56) is also zero owing to markets being segmented, i.e., home import tariff does not have any impact on the foreign firm's production meant for sales within its territory. The first term on the right-hand side of (56), which is positive, gives us the effect of a rise in import tariff at home, which leads to a rise in the domestic price of the home variety sold at home. Consequently, rents are shifted to the home firm, which now becomes more competitive and thereby drives up the domestic firm's profit. In contrast, a rise in pollution tax takes away excess profits from the home firm in terms of higher cost of production. The first-order effect of a higher pollution tax is a resultant reduction in output on account of increased cost of production (given by the third term on the right-hand side of (57)). The second-order effects (both are negative), given by the first and the second terms on the right-hand side of (57) imply that higher pollution taxes at home expand output of the foreign firm at the cost of domestic production and hence shifts rents to the foreign firm with the home firm losing its competitiveness in the international market. Hence, profit considerations for the home firm induces the government to increase domestic protection on the one hand and lower the pollution tax on the other [see Appendix B for detailed proof].

Proposition 5 *An increase in home (foreign) import tariff leads to an increase in home (foreign) firm's profit. On the contrary, home (foreign) firm's profit decline with a rise in home (foreign) pollution tax.*

2.5.3 Impact on pollution tax revenue

The effects of a change in t and τ on pollution tax revenue are

$$\frac{\partial(\tau\widetilde{X})}{\partial t} = \tau X_t = -\frac{1}{nD}\tau p_Y > 0; \tag{58}$$

for $\tau > 0$.

In case, τ is negative in equilibrium, the above effect would be negative, i.e., pollution tax revenue falls due to an increase in import tariff due to the environmental subsidy.

And,

$$\frac{\partial(\tau\widetilde{X})}{\partial\tau} = \widetilde{X} + \tau\widetilde{X}_\tau = \widetilde{X}(1 + |\lambda|); \tag{59}$$

where

$$\widetilde{X}_\tau = \frac{\partial\widetilde{X}}{\partial\tau} = X_\tau + X^*_\tau;$$

and

$$\lambda = \tau\frac{\widetilde{X}_\tau}{\widetilde{X}} < 0;$$

denotes the elasticity of output of the home firm with respect to home pollution tax, which is negative.

Now,

$$\frac{\partial(\tau\widetilde{X})}{\partial\tau}\begin{cases} > 0 \; if \; |\lambda| < 1; \\ = 0 \; f \; |\lambda| = 1; \\ < 0 \; if \; |\lambda| > 1. \end{cases} \tag{60}$$

We see that a rise in home tariff would increase pollution tax revenue for a strictly positive level of pollution tax rate. Since production of the home variety expands as a result of rise in import tariff, the revenue from pollution tax also rises for a given level of pollution tax rate. This induces the home government to raise domestic protection. On the other hand, the relation between pollution tax rate and pollution tax revenue depends on the elasticity of output with respect to pollution tax. When the percentage reduction in output exceeds the percentage rise in pollution tax, pollution tax revenue falls and vice versa.

Proposition 6(a): *Home (foreign) pollution tax revenue is increasing in home (foreign) import tariff.*

Proposition 6(b): *Home (foreign) pollution tax revenue is increasing in home (foreign) pollution tax when the absolute value of the output elasticity with respect to tax rate is less than unity and vice versa.*

2.5.4 Effect on environmental damage

Let the total environmental damage at home is given by

$$nE(\widetilde{X}) = n\theta(\widetilde{X}); \tag{61}$$

where $E(\widetilde{X})$ denotes per capita disutility from pollution at home.

The effects of a change of t and τ on total environmental damage are given as

$$\frac{\partial\Big(nE(\widetilde{X})\Big)}{\partial t} = n\theta X_t = -\frac{1}{nD}n\theta p_Y > 0; \tag{62}$$

and

$$\frac{\partial\Big(nE(\widetilde{X})\Big)}{\partial \tau} = n\theta \widetilde{X}_\tau = 2n\theta\left(\frac{1}{nD}p_X + \frac{1}{n^*D^*}p^*_{X^*}\right) < 0. \tag{63}$$

While pollution tax reduces environmental damage, an import tariff would increase damage due to rise in the production of home variety.

Proposition 7: *Environmental damage in the home (foreign) country rises with a rise in home (foreign) import tariff. In contrast, a pollution tax in the home (foreign) country reduces the same.*

2.5.5 Effect on tariff revenue

The impacts of a policy change on tariff revenue are captured by

$$\frac{\partial(tY)}{\partial t} = Y + tY_t = Y(1 + |\mu|); \tag{64}$$

where

$$\mu = t\frac{Y_t}{Y} < 0;$$

denotes the elasticity of home imports with respect to home tariff, which is negative.

Now,

$$\frac{\partial(tY)}{\partial t}\begin{cases} > 0 \; if \; |\mu| < 1; \\ = 0 \; if \; |\mu| = 1; \\ < 0 \; if \; |\mu| > 1. \end{cases} \tag{65}$$

Further,

$$\frac{\partial(tY)}{\partial \tau} = tY_\tau = -\frac{1}{nD}tp_Y > 0. \tag{66}$$

Therefore, in order that tariff revenue rises with a rise in import tariff requires that the absolute value of the elasticity of imports with respect to the tariff is less than one. When the import function is less elastic, the percentage rise in tariff exceeds the percentage fall in the imports. In case of a rise in pollution tax, there is a resultant rise in tariff revenue, for a given level of tariff. This is due to the substitution of consumption from the home variety to the foreign variety driven by the rise in prices of the former.

Proposition 8(a): *Home (foreign) tariff revenue is increasing in home (foreign) import tariff when the import demand elasticity of tariff is less than unity and vice versa.*

Proposition 8(b): *Home (foreign) tariff revenue is increasing in home (foreign) pollution tax.*

2.5.6 Characterising the non-cooperative equilibrium

By solving the first-order conditions in (41) and (42), the equilibrium import tariff, t_G and pollution tax, τ_G in the home country are derived to be [see Appendix C for detailed proof]

$$t^G = \frac{\frac{X}{n}p_X X_t + Y(\frac{dq}{dt} - 1) - (\tau - n\theta)X_t}{Y_t}; \tag{67}$$

and

$$\tau^G = n\theta + \frac{\frac{X}{n}p_X X_\tau + Y\frac{dq}{d\tau} - \frac{X^*}{n^*}p^*_{Y^*}Y^*_\tau - tY_\tau}{\widetilde{X}_\tau}; \tag{68}$$

where

$$\widetilde{X}_\tau = \frac{\partial \widetilde{X}}{\partial \tau} = \frac{\partial (X + X^*)}{\partial \tau}.$$

Similarly, the equilibrium tariff, t^*_G and pollution tax, τ^*_G in the foreign country will be

$$t^{*G} = \frac{\frac{Y^*}{n^*}q^*_{Y^*}Y^*_{t^*} + X^*(\frac{dp^*}{dt^*} - 1) - (\tau^* - n^*\theta^*)Y^*_{t^*}}{X^*_{t^*}}; \tag{69}$$

$$\tau^{*G} = n^*\theta^* + \frac{\frac{Y^*}{n^*}q^*_{Y^*}Y^*_{\tau^*} + X^*\frac{dp^*}{d\tau^*} - \frac{Y}{n}q^*_{X^*}X_{\tau^*} - t^*X^*_{\tau^*}}{\widetilde{Y}_{\tau^*}}; \tag{70}$$

where

$$\widetilde{Y}_{\tau^*} = \frac{\partial \widetilde{Y}}{\partial \tau^*} = \frac{\partial (Y + Y^*)}{\partial \tau^*}.$$

Equations (67)-(70) give us the reaction functions of home and foreign policy variables. In (67), the denominator is negative. Hence, optimal tariffs would be positive, zero or negative depending on whether the numerator is negative, zero or positive respectively. The first term of the numerator is negative and captures the rise in home firm's output owing to a rise in the domestic price following the import tariff. This is also known as the profit shifting effect (Brander and Spencer, 1985). For the import tariff to be positive, we must have $\frac{dq}{dt} \leq 1$ and $\tau \geq n\theta$. The first condition implies that the rate of change of domestic price of the imported variety with respect to the tariff must be less than or equal to one, a result similar to the one obtained by Zhang and Zhang (1998). The underlying intuition is comparable to the theory of incidence and burden-sharing of indirect taxation, which further depends on the elasticities of both demand and supply curves. Since the rise in home price of the foreign variety owing to the import tariff is shared by both home consumers and the foreign firm, the rise in price is less than or equal to the tariff. This is generally true for downward sloping demand curves and upward sloping supply curves, both of which are linear. In case, the rise in home price of imports exceeds the tariff (a situation where both demand and supply curves are downward sloping but the supply curve is flatter than the demand curve), the burden of tariff falls on the consumers to the benefit of the producer. As a result, this would pull the equilibrium tariff downwards.

In the present model, the change in domestic price of the imported variety with respect to the tariff is given by expression (B2) in Appendix B. In order to verify if this is less than unity, we compute the following:

$$q_t - 1 = \frac{1}{n^2 D}(2{p_X}^2 - {p_Y}^2) - 1. \tag{71}$$

Using the result from (30) in (71), we get

$$q_t - 1 = -\frac{1}{n^2 D}2{p_X}^2 < 0; \tag{72}$$

which shows that the rise in the domestic price of the imported variety is less than the rise in the tariff rate.

The second condition implies that the pollution tax does not fall short of the marginal damage from pollution. When the marginal damage from pollution exceeds the pollution tax, the social planner driven by the consideration for a better environmental quality encourages imports by slashing the import tariff.

Next, substituting for the terms from Table 6 [see Appendix B and Appendix C] and the expressions in (30) and (B2) simplifies the result in (67) to

$$t^G = -\frac{X}{n}\frac{p_Y}{2} - \frac{Y}{n}p_X + (\tau - n\theta)\frac{p_Y}{2p_X}. \tag{73}$$

For strictly positive levels of output, the first two terms on the right-hand side of (73) are strictly positive. Therefore, equilibrium import tariff would be positive ($t^G > 0$) only if the pollution tax is at least equal to the marginal damage from pollution, i.e., $\tau \geq n\theta$. Since the home firm would suffer a cost disadvantage from a stringent environmental regulation, the government compensates the loss in the firm's profits through an import tariff.

In (68), the denominator of the second term on the right-hand side is negative. The optimal pollution tax would be greater than or equal to the marginal damage from pollution only if the rise in tariff revenue from rise in imports driven by rise in domestic pollution tax (i.e., Y_τ) exceeds the reduction in home firm's profits from falling demand and output of the home variety and simultaneous rise in production of the foreign variety (the former is given by the first two terms and latter is given by third term in the numerator). Analogous conditions can be derived for the foreign country.

Substituting for the terms in (68) from Table 6 [see Appendix B and Appendix C] and the expressions in (30) and (31) and the expression from (B4) yield

$$\tau^G = n\theta + \frac{\frac{X}{n^2 D}p_X^2 + \frac{Y}{n^2 D}p_X p_Y + \frac{X^*}{n^{*2}D^*}{p^*_{Y^*}}^2 - t\big(-\frac{1}{nD}p_Y\big)}{\frac{1}{nD}p_X + \frac{1}{n^* D^*}p^*_{X^*}}. \tag{74}$$

Since the denominator of the second term on the right-hand side of (74) is negative, equilibrium pollution tax would exceed the marginal damage from pollution if and only if the numerator of the second term is strictly negative. For positive levels of output, the first three terms of the numerator are strictly positive. In contrast, the last term of the numerator is negative for a strictly positive level of import tariff. In case of an import subsidy, i.e., $t < 0$, equilibrium pollution tax is less than the marginal damage from pollution. An import subsidy reduces the competitiveness of the home firm and induces the government to contain the cost disadvantage of the firm through a relatively less stringent environmental regulation. A necessary condition for the equilibrium pollution tax to be at least equal to the marginal damage from pollution is that the import tariff set must have a lower bound, say $\underline{t}$. From (74) ,

$$\begin{aligned} \tau^G \geq n\theta \ \text{ if } \ \frac{X}{n^2 D}p_X^2 + \frac{Y}{n^2 D}p_X p_Y + \frac{X^*}{n^{*2}D^*}{p^*_{Y^*}}^2 - t\big(-\frac{1}{nD}p_Y\big) \leq 0; \\ \Rightarrow \tau^G \geq n\theta \ \text{ if } \ t \geq \frac{\frac{X}{n^2 D}p_X^2 + \frac{Y}{n^2 D}p_X p_Y + \frac{X^*}{n^{*2}D^*}{p^*_{Y^*}}^2}{\big(-\frac{1}{nD}p_Y\big)}; \\ \Rightarrow \tau^G \geq n\theta \ \text{ if } \ t \geq \underline{t}; \end{aligned} \tag{75}$$

where

$$\underline{t} = \frac{\frac{X}{n^2 D}p_X^2 + \frac{Y}{n^2 D}p_X p_Y + \frac{X^*}{n^{*2}D^*}{p^*_{Y^*}}^2}{\big(-\frac{1}{nD}p_Y\big)} > 0. \tag{76}$$

Given the sign restrictions, the lower bound of the import tariff is strictly positive for strictly positive levels of output. Whenever, the import tariff exceeds the lower bound, equilibrium pollution tax exceeds the marginal damage from pollution. Trade protection ensures a competitive edge to the home firm and increases domestic production. As a result, the government raises environmental regulations to combat the welfare loss from increased pollution.

The second-order conditions are verified in Appendix D.

2.5.7 Strategic policy interactions

In this section, we compute pairwise interdependencies in policies in a non-cooperative framework. The direction of change can be obtained by differentiating the first-order condition for each policy variable with respect to at most one of the policies while holding the remaining policy variables at the non-cooperative equilibrium level. Following Straume (2006) , we analyse policy interdependencies using the decomposed effects. We do this by tracing out movements in the policy variables by considering the change in the marginal effects of a particular policy change due to a change in the other policy on each component of the welfare function, namely, consumer surplus, firm's profit, pollution tax revenue, environmental damage and tariff revenue. These individual effects have been already discussed in Section 2.5. First, we consider the movement in home import tariff due to a change in home pollution tax, foreign import tariff and foreign pollution tax. In the subsequent section, we trace out the movement in home pollution tax on account of a change in home import tariff, foreign import tariff and foreign pollution tax.

2.5.8 Movement in tariff on imports in the home country

Differentiating (45) with respect to τ, τ^* and t^* yields

$$\frac{\partial}{\partial \tau}\left(\frac{\partial (nS)}{\partial t}\right) = -\frac{1}{n^3 D^2} p_Y^3 > 0; \tag{77}$$

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial (nS)}{\partial t}\right) = -\frac{1}{n^3 D^2} p_X (4p_X^2 - 3p_Y^2) > 0; \tag{78}$$

and

$$\frac{\partial}{\partial t^*}\left(\frac{\partial (nS)}{\partial t}\right) = 0. \tag{79}$$

Expression (77) shows that a rise in home pollution tax leads to a reduction in the adverse marginal effect of increased import tariff on home consumer surplus. When the home tariff is t_G, a rise in home pollution tax increases the output of the foreign firm since it generates a cost advantage owing to an increase in its rival's (home firm's) costs. This tempers the price rise of the imported variety at home and thereby represses the negative impact on consumer surplus. Therefore, domestic protection rises as environmental regulations get tougher at home. In a similar fashion, a rise in foreign pollution tax reduces the negative effect of increased home tariff on imports on home consumer surplus (given by expression (78)). As the foreign pollution tax rises, output of the home firm rises (from the result in (37)) and tempers the price rise of the home output sold in the local market. Consequently, with a rise in foreign pollution tax, the home government raises domestic protection. Finally, there are no first order effects of a change in foreign import tariff on the marginal effect of a change of home tariff on consumer surplus, since markets are segmented (given by (79)).

Now, we differentiate (56) with respect to τ, τ^* and t^* in order to find their impact on the positive effect of increased tariff on home imports on the home firm's profit.

$$\frac{\partial}{\partial \tau}\left(\frac{\partial \pi}{\partial t}\right) = \frac{1}{n^3 D^2} 4p_X^2 p_Y < 0; \tag{80}$$

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial \pi}{\partial t}\right) = -\frac{1}{n^3 D^2} 2p_X p_Y^2 > 0; \tag{81}$$

and

$$\frac{\partial}{\partial t^*}\left(\frac{\partial \pi}{\partial t}\right) = 0. \tag{82}$$

Therefore, a rise in pollution tax at home lowers the positive marginal effect of tariff on home profits (given by (80)). Higher pollution tax lowers domestic output and hence reduces the increased competitiveness of the home firm from a rise in home import tariff. In order that the firm is able to retain its market power in the international market, import tariffs must rise. On the contrary, from (81) we can see that the home firm becomes more competitive with a rise in foreign pollution tax. Therefore, the home government's incentive to capture foreign rents reduces and hence home tariff falls. Furthermore, (82) shows that the import tariff imposed by the foreign country does not have any direct impact on the marginal effect of a change of home tariff on the home firm's profit.

We now turn to the impacts of a change of τ, τ^* and t^* on the change in pollution tax revenue, at margin, due to increased home tariff. Differentiating (58) with respect to τ, τ^* and t^* yields

$$\frac{\partial}{\partial \tau}\left(\frac{\partial (\tau \widetilde{X})}{\partial t}\right) = -\frac{1}{nD} p_Y > 0; \tag{83}$$

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial (\tau \widetilde{X})}{\partial t}\right) = 0; \tag{84}$$

and

$$\frac{\partial}{\partial t^*}\left(\frac{\partial (\tau \widetilde{X})}{\partial t}\right) = 0. \tag{85}$$

From (83), it is evident that a higher pollution tax at home increases the marginal revenue from pollution tax due to increased import tariff. When an import tariff is imposed on the imported variety, output of the home variety rises, thereby raising the marginal revenue from increased pollution tax. Hence, the incentive to capture foreign rents falls, which results in a lower import tariff at home. Expressions (84) and (85) show that any change in the home pollution tax revenue owing to a rise in the level of domestic protection is not directly driven by changes in the foreign policies.

In order to compute the effect of a change in τ, τ^* and t^* on the marginal effect of change of import tariff at home on the overall environmental damage at home, we differentiate the expression in (62) with respect to the above policy variables. We get

$$\frac{\partial}{\partial \tau}\left(\frac{\partial(nE(\widetilde{X}))}{\partial t}\right) = \frac{\partial}{\partial \tau^*}\left(\frac{\partial(nE(\widetilde{X}))}{\partial t}\right) = \frac{\partial}{\partial t^*}\left(\frac{\partial(nE(\widetilde{X}))}{\partial t}\right) = 0; \tag{86}$$

which shows that there is no impact of a change in home pollution tax on the marginal increase in environmental damage from the imposition of a tariff. This is due to the assumption of linear demand functions. So some positive marginal effects must be fully offset by negative marginal effects such that this remains unchanged. For instance, a rise in home import tariffs increases total environmental damage owing to expansion of production of the home firm. However, this effect is completely offset by a rise in home pollution tax. Further, foreign policies have no direct impact on the marginal effect of import tariff on environmental damage.

Lastly, we differentiate (64) with respect to τ, τ^* and t^* for computing their effect on the change in home tariff revenue from an increase in domestic protection. Differentiating (64) with respect to τ, τ^* and t^* yields

$$\frac{\partial}{\partial \tau}\left(\frac{\partial(tY)}{\partial t}\right) = -\frac{1}{nD}p_Y > 0; \tag{87}$$

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial(tY)}{\partial t}\right) = \frac{1}{nD}2p_X < 0; \tag{88}$$

and

$$\frac{\partial}{\partial t^*}\left(\frac{\partial(tY)}{\partial t}\right) = 0. \tag{89}$$

A rise in home pollution tax increases the marginal effect of a rise in import tariff on tariff revenue (shown by (87)). When the tariff revenue increases (decreases) with tariff (a situation where imports are less (more) elastic with respect to a tariff), a rise in home pollution tax increases (reduces) this positive (negative) effect. Hence, consideration for tariff revenue would be reflected in lower (higher) domestic protection. Imports being less elastic with respect to the import tariff, a substantial rise in import tariff leads to a less than proportionate reduction in imports and hence increases tariff revenue (see condition (65)). From (87), this positive marginal effect is aggravated by a rise in home pollution tax. Therefore, the government is induced to reduce home protection when home pollution tax rises. The converse is true when imports are highly elastic with respect to import tariff. In contrast, expression (88) shows that a rise in foreign pollution tax leads to a rise (fall) in home import tariff when the elasticity of imports with respect to the tariff is less (more) than one. Finally, the result in (89) shows that foreign tariffs do not have any direct impact on the marginal effect of import tariffs at home on home tariff revenue due to the assumption of segmented markets.

2.5.9 Movement in home pollution tax

In this section, we compute the impact of change of home tariffs, foreign tariffs and foreign pollution tax on home pollution tax.

Differentiating equation (46) with respect to t, t^* and τ^* yields

$$\frac{\partial}{\partial t}\left(\frac{\partial(nS)}{\partial \tau}\right) = -\frac{1}{n^3D^2}p_X(4p_X^2 - 3p_Y^2) > 0; \tag{90}$$

$$\frac{\partial}{\partial t^*}\left(\frac{\partial(nS)}{\partial \tau}\right) = 0; \tag{91}$$

and

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial (nS)}{\partial \tau}\right) = -\frac{1}{n^3 D^2} p_Y^3 > 0. \tag{92}$$

From expression (90), we see that the environmental policy gets more stringent with an increase in domestic protection. As the import tariff at home rises, the resultant rise in domestic production offsets the negative marginal effect on consumer surplus due to the imposition of a pollution tax on domestic production. In contrast, an import tariff imposed by the foreign country has no impact on this negative effect (expression (91)). This is driven by the assumption of separation of markets, i.e., a foreign trade policy has no effect on domestic prices. Expression (92) shows that home pollution tax is increasing in foreign pollution tax. As the foreign pollution tax rises, the home firm faces a marginal cost advantage relative to its rival which offsets the price rise of the output at home. This leads to a reduction in the negative effect of home pollution tax on home consumer surplus.

Now, we differentiate equation (57) with respect to t, t^* and τ^* in order to find their impact on the marginal effect of increased pollution tax at home on the home firm's profit. This exercise gives us the following results:

$$\frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial \tau}\right) = \frac{1}{nD} p_Y \left(1 + \frac{1}{n^2 D} p_Y^2\right) > 0; \tag{93}$$

$$\frac{\partial}{\partial t^*}\left(\frac{\partial \pi}{\partial \tau}\right) = -\frac{1}{{n^*}^3 {D^*}^2} 8 p^*_{X^*} > 0; \tag{94}$$

and

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial \pi}{\partial \tau}\right) = 4\Big(\frac{1}{n^3 D^2} p_X^2 p_Y + \frac{1}{{n^*}^3 {D^*}^2} {p^*}^2_{X^*} p^*_{Y^*}\Big) < 0. \tag{95}$$

Expression (93) shows that an increase in domestic protection results in a tougher environmental policy at home. In general, an increase in home import tariff leads to an expansion in the output sold in the local market. This rise in output would temper the negative effect on the home firm's profit due to imposition of the pollution tax. In case of a rise in foreign import tariff, the subsequent reduction in home exports compensates for the negative effect of the pollution tax at home on the home firm's profit (given by expression (94)). Therefore, home pollution tax rises with foreign import tariff. From (95), it is evident that home pollution tax decreases with a rise in foreign pollution tax. As the environmental policy in the foreign country tightens, the negative effect on the home firm's profit exacerbates owing to a fall in marginal revenue in both home and foreign markets due to a rise in its output.

We now differentiate equation (59) with respect to t, t^* and τ^* to examine the impact on the marginal effect of home pollution tax revenue due to an increase in the pollution tax at home. We get the following expressions:

$$\frac{\partial}{\partial t}\left(\frac{\partial (\tau \widetilde{X})}{\partial \tau}\right) = -\frac{1}{nD} p_Y > 0; \tag{96}$$

$$\frac{\partial}{\partial t^*}\left(\frac{\partial (\tau \widetilde{X})}{\partial \tau}\right) = \frac{1}{n^* D^*} 2 p^*_{X^*} < 0; \tag{97}$$

and

$$\frac{\partial}{\partial \tau^*}\left(\frac{\partial (\tau \widetilde{X})}{\partial \tau}\right) = -\Big(\frac{1}{nD} p_Y + \frac{1}{n^* D^*} p^*_{Y^*}\Big) > 0. \tag{98}$$

From (96), we see that a rise in home import tariff raises the marginal effect of a rise in home pollution tax on the tax revenue. In case, the elasticity of home output is less than one, the rise in import tariff is accompanied by a fall in home pollution tax. This is because of the fact that the revenue from pollution tax is increasing with the tax and this positive effect is further amplified by a rise in home import tariff. Otherwise, pollution tax is non-decreasing in home import tariff. When the level of protection rises in the foreign country, the marginal effect on pollution tax revenue falls (given by expression (97)). When the domestic output is less elastic with respect to a pollution tax, the revenue from pollution tax would be increasing. However, with a rise in foreign protection, production of export, X^* falls, which reduces the pollution tax revenue at margin. Consequently, considerations for tax revenue pull in the direction of a higher pollution tax rate. This relationship reverses when the revenue from pollution tax falls with a rise in the tax rate. In this case, a higher import tariff abroad is accompanied by a less stringent environmental policy at home. Lastly, expression (98) shows that home pollution tax would fall (rise) with a rise in foreign pollution tax when the tax revenue from pollution at home is increasing (decreasing) with the tax rate.

The next step is to examine the relationship between policy variables t, t^* and τ^* and the negative effect on marginal damage from increased taxation on pollution. We differentiate equation (63) with respect to the above variables to obtain the following:

$$\frac{\partial}{\partial t}\left(\frac{\partial(nE(\widetilde{X}))}{\partial\tau}\right)=\frac{\partial}{\partial t^*}\left(\frac{\partial(nE(\widetilde{X}))}{\partial\tau}\right)=\frac{\partial}{\partial\tau^*}\left(\frac{\partial(nE(\widetilde{X}))}{\partial\tau}\right)=0; \tag{99}$$

which shows that this effect is independent of any change in t, t^* and τ^*. This is again due to the assumption of linear demand curves.

Finally, we differentiate equation (66) with respect to t, t^* and τ^* in order to find the impact on the positive effect of increased pollution tax on home tariff revenue. The results are as follows:

$$\frac{\partial}{\partial t}\left(\frac{\partial(tY)}{\partial\tau}\right)=-\frac{1}{nD}p_Y>0; \tag{100}$$

and

$$\frac{\partial}{\partial t^*}\left(\frac{\partial(tY)}{\partial\tau}\right)=\frac{\partial}{\partial\tau^*}\left(\frac{\partial(tY)}{\partial\tau}\right)=0. \tag{101}$$

We notice that a rise in home import tariff increases the positive effect of increased pollution tax on home tariff revenue (given by expression (100)). Therefore, tariff revenue considerations would tend to pull the pollution tax downwards. Evidently, the incentive to raise pollution tax to expropriate the additional revenue from tariff keeps falling as home tariff rises. Expression (101) shows that foreign policies have no impact on the gain in home tariff revenue owing to the linearity assumption.

2.5.10 Best response functions measuring the overall effects

Apart from the decomposed marginal effects discussed in the previous section, we need to measure the total effect of a change of one policy on any other. This is obtained by total differentiation of the first-order conditions for the social optimum (given by the conditions in (41) and (42)).

We begin with the first-order condition for the non-cooperative equilibrium import tariff at home (condition (41)) to derive the best response function of home import tariff. Using results from Section 2.5.1-Section 2.5.5, condition (41) can be expanded as

$$W_t^G=\Big(-(Xp_t+Yq_t)\Big)+\Big(\frac{X}{n}p_YY_t\Big)+\Big(\tau X_t\Big)+\Big(-n\theta X_t\Big)+\Big(Y+tY_t\Big)=0. \tag{102}$$

Differentiating (102) totally and utilising expressions from (D1), (D5) and (D6) [see Appendix D] yield

$$W_{tt}^G dt+W_{t\tau}^G d\tau+W_{t\tau^*}^G d\tau^*=0; \tag{103}$$

where

$$W_{t\tau^*}^G=-(p_tX_{\tau^*}+q_tY_{\tau^*})+\frac{1}{n}p_YY_tX_{\tau^*}+Y_{\tau^*}; \tag{104}$$

is obtained by partially differentiating (102) with respect to τ^* [see Appendix E for a detailed proof].

Re-arranging terms in (103) and using the results from (29) and (32)-(39) give us

$$dt=\lambda_1^G d\tau+\lambda_2^G\tau^*; \tag{105}$$

where,

$$\lambda_1^G=-\frac{W_{t\tau}^G}{W_{tt}^G}=\frac{1}{3p_X}p_Y>0; \tag{106}$$

and

$$\lambda_2^G=-\frac{W_{t\tau^*}^G}{W_{tt}^G}=-\frac{1}{3}<0; \tag{107}$$

which implies that the incentive for the home government to raise home import tariff outweighs the disincentive from the same [as discussed in Section 2.5.8 and subsequently summarised in Table 1(a)]. Hence, increased domestic protection is accompanied with a tougher environmental policy at home (from (106)). In contrast, a rise in foreign pollution tax is accompanied by a reduction in home protection. A tougher environmental policy at home strengthens the rent-shifting incentive of the home government which leads to an increase in domestic protection. Higher pollution taxes at home have an adverse impact on the profits of the home firm in comparison to the foreign firm. Further, consumers at home are worse-off owing to a rise in domestic price of the home firm's output. However, there are welfare gains from an improvement in environmental quality due to stricter regulations on production activities generating pollution. The converse is true when the foreign country raises its pollution tax (from (107)). From (105), we notice that domestic trade policy is independent of foreign trade policy. This is due to the assumption of segmented markets.[5] However, home import tariff depends on foreign import tariff via the reaction functions for home and foreign pollution tax.

Proposition 9: *Home (foreign) tariff increases when:*

(i) the home (foreign) country undertakes a tougher environmental policy;

(ii) the foreign (home) country undertakes a laxer environmental policy.

Also, foreign (home) tariff has no first-order effect on home (foreign) tariff.

Next, we derive the best response of home pollution tax. Using results in Section 2.5.1-Section 2.5.5, condition (42) can be re-written as

$$W^G_\tau = \Big(-(Xp_\tau + Yq_\tau)\Big) + \Big(\frac{X}{n}p_Y Y_\tau + \frac{X^*}{n^*}p^*{}_{Y^*}Y^*_\tau - \widetilde{X}\Big) + \Big(\widetilde{X} + \tau\widetilde{X_\tau}\Big) + \Big(-n\theta\widetilde{X_\tau}\Big) + \Big(tY_\tau\Big) = 0. \tag{108}$$

Total differentiation of (108) and further substitution of expressions (D2) and (D5) [see Appendix D] yield

$$W^G_{\tau\tau}d\tau + W^G_{\tau t}dt + W^G_{\tau t^*}dt^* + W^G_{\tau\tau^*}d\tau^* = 0; \tag{109}$$

where

$$W^G_{\tau t^*} = \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{t^*} = -\frac{1}{n^{*3}D^{*2}}2p^*_{X^*}p^{*2}_{Y^*} > 0; \tag{110}$$

and

$$W^G_{\tau\tau^*} = -(p_\tau X^*_\tau + q_\tau Y^*_\tau) + \frac{1}{n}p_Y Y_\tau X^*_\tau + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{\tau^*} = \frac{1}{n^{*3}D^{*2}}p*^3_{Y^*} < 0; \tag{111}$$

are derived by partially differentiating (108) with respect to t^* and τ^*, respectively.

Re-arranging terms in (109) and using results in (29) and (32)-(39) give us the following result:

$$d\tau = \Lambda^G_1 dt + \Lambda^G_2 dt^* + \Lambda^G_3 d\tau^*; \tag{112}$$

where,

$$\Lambda^G_1 = -\frac{W^G_{\tau t}}{W^G_{\tau\tau}} = -\frac{\overbrace{-\frac{1}{n^3D^2}p_Y(4p^2_X - p^2_Y)}^{(+)}}{\underbrace{\frac{1}{n^3D^2}p_X(4p^2_X - p^2_Y) + \frac{1}{n^{*3}D^{*2}}4p^*_{X^*}(2p^{*2}_{X^*} - p^{*2}_{Y^*})}_{(-)}} > 0; \tag{113}$$

$$\Lambda^G_2 = -\frac{W^G_{\tau t^*}}{W^G_{\tau\tau}} = -\frac{\overbrace{-\frac{1}{n^{*3}D^{*2}}2p^*_{X^*}p^{*2}_{Y^*}}^{(+)}}{\underbrace{\frac{1}{n^3D^2}p_X(4p^2_X - p^2_Y) + \frac{1}{n^{*3}D^{*2}}4p^*_{X^*}(2p^{*2}_{X^*} - p^{*2}_{Y^*})}_{(-)}} > 0; \tag{114}$$

[5] In case markets are inter-connected, any policy distortion introduced by the home government that regulates foreign output meant for sales within the domestic territory (say, an import tariff) would affect the foreign output sold in the foreign market.

and

$$\Lambda_3^G = -\frac{W^G_{\tau\tau^*}}{W^G_{\tau\tau}} = -\frac{\overbrace{\frac{1}{n^{*3}D^{*2}}p*^3_{Y^*}}^{(-)}}{\underbrace{\frac{1}{n^3D^2}p_X(4p_X^2 - p_Y^2) + \frac{1}{n^{*3}D^{*2}}4p^*_{X^*}(2p^{*2}_{X^*} - p^{*2}_{Y^*})}_{(-)}} < 0. \tag{115}$$

and the signs of the above coefficients follow from conditions (5), (29)-(31), (D2), (D5), (110) and (111).

Equation (112) shows that home pollution tax is increasing in both home and foreign import tariff but decreasing in foreign pollution tax. The incentive to raise home pollution tax outweighs the incentive to reduce the same when domestic protection rises. The incentives for a higher pollution tax include: *one,* an improved the environmental quality; *two,* higher revenue from pollution tax provided that the output is less responsive to a rise in the pollution tax rate; three, an increase in import tariff revenue as consumers switch from the costly home variety to imports. In contrast, the disincentives from a higher pollution tax are: *one,* higher domestic prices causing a loss in consumer surplus; *two,* lower profits for the firm; and, *three,* reduction in pollution tax revenue when output is highly elastic to changes in the pollution tax rate. Now, increased domestic protection improves the competitiveness of the home firm and also reduces the home government's incentive to capture rents from the foreign country as a consequence of strategic trade. As a result, environmental policy becomes more stringent. When the foreign import tariff rises, the negative marginal effect of increased pollution tax on the home firm's profit falls. This is due to the reduction in the home output meant for exports which moderates: (a) the effect of decreased marginal revenue from sales of home output in the foreign country; (b) the cost disadvantage from increased taxation on pollution. Therefore, home pollution tax would rise. However, considerations for pollution tax revenue alone would pull home pollution tax downward (upward) depending on the elasticity of output with respect to pollution tax being more (less) than one. From (114), we know that a rise in foreign protection decreases the marginal gain in tax revenue due to a fall in home exports. In the present scenario, profit considerations for the home firm outweigh the considerations for pollution tax revenue. As a result, home pollution tax rises with foreign protection. Lastly, home pollution tax is inversely related to foreign pollution tax (expression (115)). While considerations for consumer surplus from a rise in foreign pollution tax lead to a rise in home pollution tax, this effect is outweighed by the consideration for home firm's profit and pollution tax revenue (when it is increasing in pollution tax rate). A rise in foreign pollution tax leads to an expansion of home output and aggravates the negative marginal effect of a rise in home pollution tax on the home firm's profit (expression (95)). This leads to a reduction in home pollution tax. Home pollution tax also moves downward (upward) when pollution tax revenue is increasing (decreasing) in the tax rate, i.e., output is less (more) elastic to changes in tax rate. The net effect of the two factors which pulls the tax downward, outweighs the effect which pulls it in the upward direction owing to a marginal gain in consumer surplus from an increase in foreign pollution tax (from expression (92). Hence, the home government sets a less stringent environmental policy. Table 1(b) summarises these results.

Proposition 10: *Home (foreign) pollution tax increases when:*

(i) the home (foreign) country raises its import tariff;

(ii) the foreign (home) country raises its import tariff.

(iii) the foreign (home) country reduces pollution tax.

Quite contrary to the interaction between the import tariff at home and pollution taxes at home and abroad, the expressions in (79), (82), (85), (86) and (89) show that the tariff on imports by the foreign country do not have any direct impact on domestic protection. Since markets are segmented, import tariffs of one trading nation do not appear directly into the best response function of its partner. Instead, strategic interaction in trade policies enters in an implicit form through the reaction functions for environmental policies, i.e.,

$$t = t(\tau, \tau^*);\ t^* = t^*(\tau, \tau^*); \tag{116}$$

where

$$\tau = \tau(t, t^*, \tau^*);\ \tau^* = \tau^*(t, t^*, \tau). \tag{117}$$

Here, (116) denotes the set of best response functions for trade policies of the two countries. These are composite functions, which further depend on the set of equations given in (117).

Therefore, the response of t with respect to a change in t^* is given by

$$\left.\frac{dt}{dt^*}\right|_G = \left.\frac{dt}{d\tau}\right|_G . \left.\frac{d\tau}{dt^*}\right|_G = \lambda_1^G . \Lambda_2^G > 0; \tag{118}$$

where subscript G or superscript G represents the effects pertaining to the social optimum.

Expression (118) shows that the change in foreign tariff generates a second-order effect on home tariff via the change in home pollution tax. Table 1(a) and Table 1(b) summarise all of the above results.

Table 1(a): Response of home import tariff on the components of aggregate social welfare to changes in policy variables

Policy variables	Consumer surplus	Firm's profit	Pollution tax revenue	Environmental damage	Import tariff revenue	Overall response of t
τ	(+)	(+)	(-)	0	(+)/(-)*	(+)
τ^*	(+)	(-)	0	0	(+)/(-)*	(-)
t^*	0	0	0	0	0	(+)

* The sign depends on the elasticity of import demand with respect to home import tariff, as stated in (65), (87) and (88).

Table 1(b): Response of home pollution tax on the components of aggregate social welfare to changes in policy variables

Policy variables	Consumer surplus	Firm's profit	Pollution tax revenue	Environmental damage	Import tariff revenue	Overall response of τ
t	(+)	(+)	(+)/(-)**	0	(-)	(+)
t^*	0	(+)	(+)/(-)**	0	0	(+)
τ^*	(+)	(-)	(+)/(-)**	0	0	(-)

** The sign depends on the elasticity of output with respect to home pollution tax, as stated in (60), (96)-(98).

Figure 1 and Figure 2 show the best response functions for each pair of policies, which are derived from conditions that ensure stability of the equilibria [see Table 2 . Each of the stability conditions show the respective slopes of the reaction functions of a pair of policies in each quadrant. This follows from expressions (106), (107) and (113)-(115). The reaction functions ϕ_i (and ϕ_i^*) and ψ_i (and ψ_i^*) represent the best response of home (foreign) import tariff and home (foreign) pollution tax, respectively. The subscripts $i = 1,\ 2$ for ϕ_i (and ϕ_i^*) and ψ_i (and ψ_i^*) refer to the arguments of the best response functions pertaining to trade and environmental policies at home and foreign, respectively. Finally, the reaction functions θ (and θ^*) denote the best response of home (foreign) import tariff to foreign (home) import tariff and functions Θ (and Θ^*) denote the best response of home (foreign) pollution tax to foreign (home) pollution tax.

Table 2: Stability conditions

Quadrant	Plane	Condition†
I	(t, τ)	$W_{tt}^G W_{\tau\tau}^G > W_{t\tau}^G W_{\tau t}^G$
	(t, t^*)	$W_{tt}^G W_{t^*t^*}^G > W_{tt^*}^G W_{tt^*}^G$
II	(t^*, τ)	$W_{t^*t^*}^G W_{\tau\tau}^G > W_{t^*\tau}^G W_{\tau t^*}^G$
III	(t^*, τ^*)	$W_{t^*t^*}^G W_{\tau^*\tau^*}^G > W_{t^*\tau^*}^G W_{\tau^*t^*}^G$
	(τ, τ^*)	$W_{\tau\tau}^G W_{\tau^*\tau^*}^G > W_{\tau\tau^*}^G W_{\tau^*\tau}^G$
IV	(t, τ^*)	$W_{tt}^G W_{\tau^*\tau^*}^G > W_{t\tau^*}^G W_{\tau^*t}^G$

† These conditions are associated with the slopes of the respective reaction functions which ensure stability, i.e., they ensure that the Hessian matrix is positive definite (a sufficient condition for local maximum).

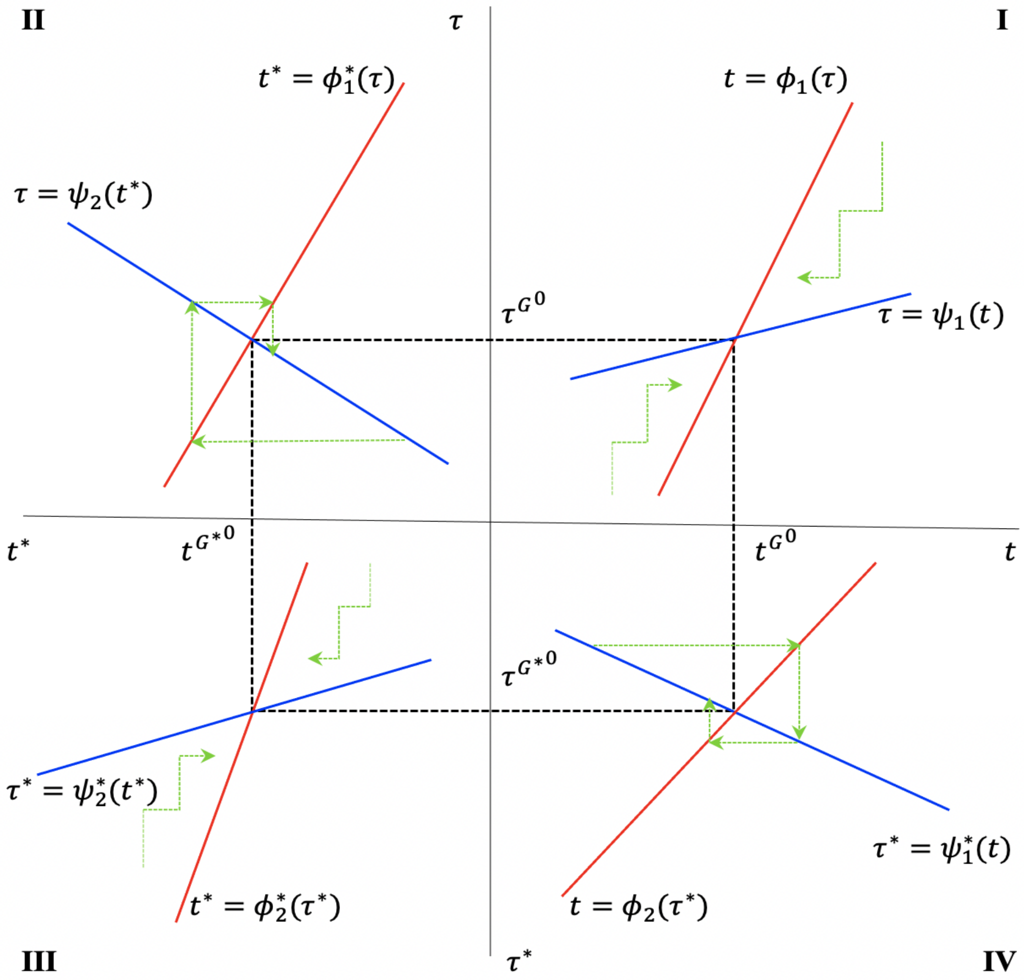


Figure 1: Trade and environmental policy interaction

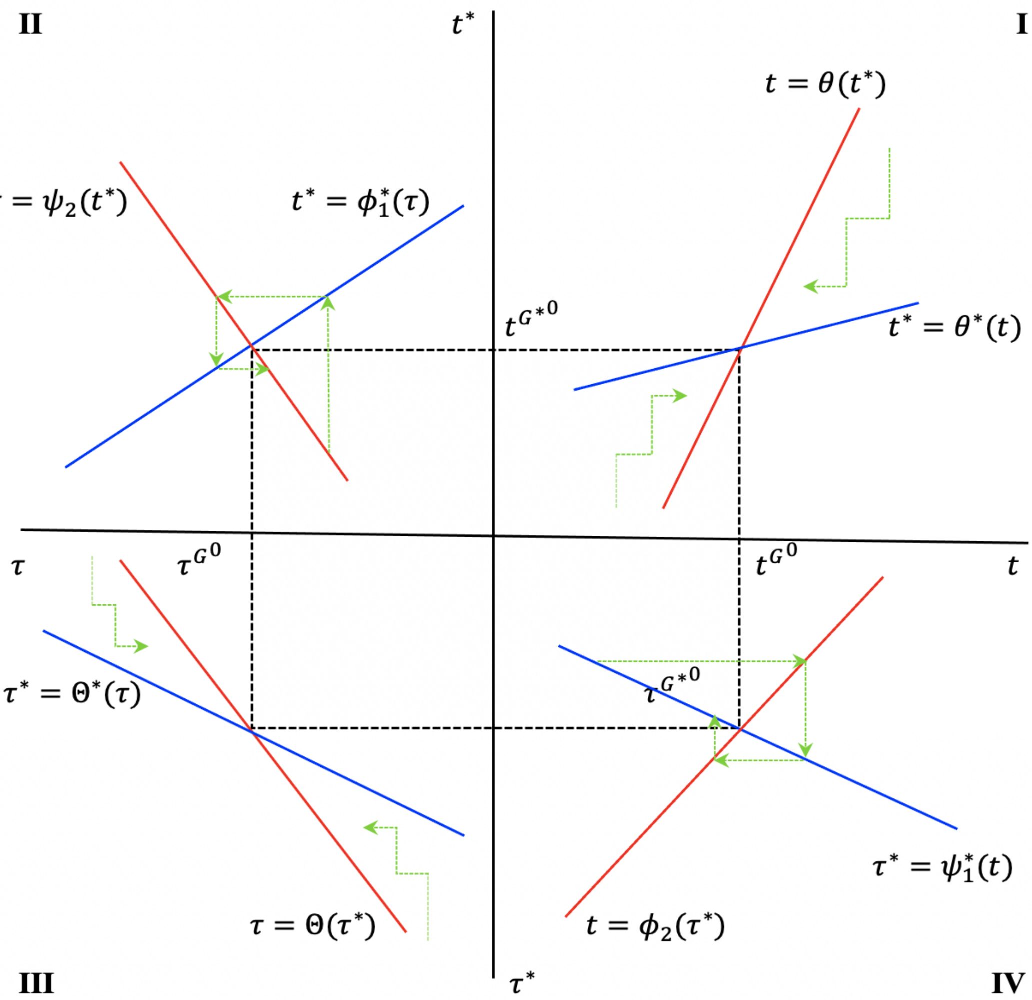


Figure 2: Trade policy interaction and environmental policy interaction

2.6 The political equilibrium

The two policy instruments, namely import tariffs and pollution tax, are employed by the policy maker to control imports and local pollution respectively. Firms in each country are susceptible to any change in the policy paradigm. Moreover, the firms being imperfectly competitive, have an incentive to use the positive economic profit to sway policies in their favour. Each firm acts as a producer lobby which presses the policy maker to raise import tariffs and lower pollution taxes. The policy maker or the regulator, in turn, is politically motivated and aims to maximise its political welfare. Political welfare is contingent upon the level of financial support extended by special interest groups which can be used for political advertising for motives of being re-elected.

The stages of the game are as follows. In the first stage, the lobby offers the policy maker a contribution schedule which is contingent upon the policy stance. In the next stage, the policy maker determines the optimal policy and accepts the contribution from the lobby. The firm along with the policy maker assume that foreign policies are given. In the last stage, production and consumption decisions are made and all markets clear. The game is solved through backward induction.

The utility function (gross of contributions) of the producer lobby in the home country is given as

$$B(t, \tau; t^*, \tau^*) \equiv \pi(.). \tag{119}$$

Following Grossman and Helpman (1995), the policy maker is assumed to maximise a weighted sum of the bribe received and the aggregate gross of contributions social welfare equal to

$$W^P = C(t,\tau;t^*,\tau^*) + aW^G(t,\tau;t^*,\tau^*);\ a > 0; \tag{120}$$

where $C(t,\tau;t^*,\tau^*)$ denotes the contribution schedule and a is the weight attached by the policy maker to aggregate social welfare relative to the weight on the bribe received from the producer lobby.

Based on the Bernheim and Whinston (1986) formulation, the Nash equilibrium tariff and pollution tax, t^P and τ^P can be found using the following two necessary conditions:

$$\chi = \arg\max_{\chi} C(.) + aW^G(.); \tag{121}$$

$$\chi = \arg\max_{\chi} [B(.) - C(.)] + [C(.) + aW^G(.)]. \tag{122}$$

Condition (121) requires that the equilibrium policy vector, $\chi^0 = ({t^P}^0, {\tau^P}^0)$, maximises the policy maker's utility function while by condition (122) the equilibrium policy vector also maximises the joint utility of the lobby and the policy maker.

The equilibrium characterisation is found by taking the first-order conditions for (121) and (122) which yield

$$\frac{\partial W^P}{\partial \chi} = \frac{\partial C(.)}{\partial \chi} + a\frac{\partial W^G(.)}{\partial \chi} = 0; \tag{123}$$

and

$$\left[\frac{\partial B(.)}{\partial \chi} - \frac{\partial C(.)}{\partial \chi}\right] + \left[\frac{\partial C(.)}{\partial \chi} + a\frac{\partial W^G(.)}{\partial \chi}\right] = 0. \tag{124}$$

Using (123) in (124), we have

$$\frac{\partial B(.)}{\partial \chi} = \frac{\partial C(.)}{\partial \chi}. \tag{125}$$

Equation (125) reflects the local truthfulness of the contribution schedule offered by the producer lobby.

The characterisation of the equilibrium policy vector is found by substituting (125) in (123) which yields

$$\frac{\partial W^P}{\partial \chi} = \frac{\partial B(.)}{\partial \chi} + a\frac{\partial W^G(.)}{\partial \chi} = 0. \tag{126}$$

In a similar fashion, the above first-order conditions can be decomposed into a number of components. In addition to the components discussed in case of the social optimum, any change in the weighted welfare function brought about by a change in t or τ would affect the utility function (or rent) of the producer lobby. Since the utility function is equivalent to the profit function of the home (and foreign) firm, any change of policy which affects the profit function would translate into a change in electoral contributions to the policy maker. Mathematically,

$$\frac{\partial B}{\partial t} = \frac{\partial \pi}{\partial t} = \frac{X}{n} p_Y Y_t = \frac{X}{n^2 D} 2p_X p_Y > 0; \tag{127}$$

$$\frac{\partial B}{\partial \tau} = \frac{\partial \pi}{\partial \tau} = \frac{X}{n} p_Y Y_\tau + \frac{X^*}{n^*} {p^*}_{Y^*} Y^*_\tau - \widetilde{X} = -\left(\frac{X}{n^2 D} p_Y^2 + \frac{X^*}{{n^*}^2 D^*} {p^*}^2_{Y^*} + \widetilde{X}\right) < 0. \tag{128}$$

Expressions (127) and (128) show that the welfare of the producer lobby in the home (foreign) country is rising in import tariff and declining in pollution tax at home (foreign). A rise in home (foreign) import tariff increases the domestic price of the importable in the home (foreign) country and augments competitiveness of the home (foreign) firm. This increases the profit of the home (foreign) firm. On the contrary, a rise in home (foreign) pollution tax escalates production costs incurred by the home (foreign) firm and makes the respective firm less competitive. This affects its profits adversely.

Combining this with Proposition 5, we have the following result:

Proposition 11: *Rents reaped by the home (foreign) lobby are rising in home (foreign) import tariff and decreasing in home (foreign) pollution tax.*

Next, we solve for the non-cooperative levels of import tariff and pollution tax for the home country in the presence of the producer lobby.

Employing condition (126) for $\chi = \{t, \tau\}$ and using results from (127) and (128), we derive the equilibrium levels of tariff and pollution tax at home as

$$t^P = t^G + \left(-\frac{1}{a}\frac{X}{n} p_Y \right); \tag{129}$$

where

$$t^G = \frac{\frac{X}{n} p_X X_t + Y(\frac{dq}{dt} - 1) - (\tau - n\theta) X_t}{Y_t}; \quad \text{[from (67)]}$$

and

$$\tau^P = \tau^G + \frac{1}{a\widetilde{X}_\tau} \left(\widetilde{X} - \frac{X}{n} p_Y Y_\tau - \frac{X^*}{n^*} p^*{}_{Y^*} Y^*_\tau \right); \tag{130}$$

where

$$\tau^G = n\theta + \frac{\frac{X}{n} p_X X_\tau + Y \frac{dq}{d\tau} - \frac{X^*}{n^*} p^*{}_{Y^*} Y^*_\tau - t Y_\tau}{\widetilde{X}_\tau}. \quad \text{[from (68)]}$$

Similarly, equilibrium tariff and pollution tax in the foreign country are given as

$$t^{*P} = t^{*G} + \left(-\frac{1}{a^*}\frac{Y^*}{n^*} q^*{}_{X^*} \right); \tag{131}$$

where

$$t^{*G} = \frac{\frac{Y^*}{n^*} q^*{}_{Y^*} Y^*{}_{t^*} + X^*(\frac{dp^*}{dt^*} - 1) - (\tau^* - n^*\theta^*) Y^*{}_{t^*}}{X^*_{t^*}}; \quad \text{[from (69)]}$$

and

$$\tau^{*P} = \tau^{*G} + \frac{1}{a^*\widetilde{Y}_{\tau^*}} \left(\widetilde{Y} - \frac{Y}{n} q_X X_{\tau^*} - \frac{Y^*}{n^*} q^*{}_{X^*} X^*{}_{\tau^*} \right); \tag{132}$$

where

$$\tau^{*G} = n^*\theta^* + \frac{\frac{Y^*}{n^*} q^*{}_{Y^*} Y^*{}_{\tau^*} + X^* \frac{dp^*}{d\tau^*} - \frac{Y}{n} q^*{}_{X^*} X_{\tau^*} - t^* X^*{}_{\tau^*}}{\widetilde{Y}_{\tau^*}}. \quad \text{[from (70)]}$$

Expressions (129)-(132) are the equilibrium levels of policy variables when the policy maker is politically motivated. These can be easily compared to the socially optimal levels of the policy variables in (67)-(70), respectively. In (129), the second term on the right-hand side is strictly positive [see (127)]. Hence, the optimal import tariff when firm lobbies are fully functional is higher than the one associated with the social optimum, i.e.,

$$|t^P| \geq |t^G|.$$

On the other hand, the equilibrium pollution tax is lower than the socially optimal value. This follows from the second term on the right-hand side of (130), which is unambiguously negative [see (128)]. We have

$$|\tau^P| \leq |\tau^G|.$$

Similar results hold for the foreign country.

It is noticed that imperfectly competitive firms may influence policy makers to set policies which are more favourable to them. While the higher level of protection provides a competitive edge to the firm which produces a variant of the imported good, the relatively lax environmental policy allows a cost advantage. These results allude to the studies by Damania et al. (2003) and Mehra (2010).

Proposition 12 *In case of the political equilibrium, the import tariff is higher than what is socially optimal. On the contrary, the corresponding level of pollution tax is much lower than that of the socially optimal level.*

Proposition 12 can be utilised to the explain the role of product differentiation in lobbying. From (129) and (130), we get

$$t^P - t^G = -\frac{1}{a}\frac{X}{n} p_Y; \tag{133}$$

and

$$\tau^G - \tau^P = -\frac{1}{a\widetilde{X}_\tau} \left(\widetilde{X} - \frac{X}{n} p_Y Y_\tau - \frac{X^*}{n^*} p^*{}_{Y^*} Y^*_\tau \right). \tag{134}$$

Differentiating equations (133) and (134) partially with respect to $-p_Y$, which captures the degree of product differentiation, yields

$$\frac{\partial(t^P - t^G)}{\partial(-p_Y)} = \frac{1}{a}\frac{X}{n} > 0; \tag{135}$$

and

$$\frac{\partial(\tau^G - \tau^P)}{\partial(-p_Y)} = -\frac{1}{a\widetilde{X}_\tau}\frac{X}{n}Y_\tau > 0. \tag{136}$$

Expressions (135) and (136) show that product differentiation reduces the wedge between the equilibrium levels of policy outcomes under the social and political equilibrium. As the imported variety becomes highly differentiated (i.e., $-p_Y$ falls), the domestic firm faces less competition from its foreign rival which sufficiently weakens its incentive to lobby for higher protection and lower environmental regulations.

Proposition 13: *A higher degree of product differentiation compresses the deviation between the non-cooperative equilibrium outcomes under the social optimum and the political equilibrium.*

Furthermore, the wedge between the two outcomes is also driven by the level of corruption (given by a fall in a). Differentiating (129) and (130) with respect to the parameter a, we get

$$\frac{\partial t^P}{\partial a} = \frac{1}{a^2}\frac{X}{n}p_Y < 0; \tag{137}$$

and

$$\frac{\partial \tau^P}{\partial a} = -\frac{1}{a^2\widetilde{X}_\tau}\left(\widetilde{X} - \frac{X}{n}p_Y Y_\tau - \frac{X^*}{n^*}{p^*}_{Y^*}Y^*_\tau\right) > 0. \tag{138}$$

In the political equilibrium, optimal tariff level rises with a fall in the weight attached aggregate social welfare (i.e., the policy maker becomes more corrupt). In contrast, the optimal pollution tax is rising in a, i.e., increased corruption lowers pollution tax.

Proposition 14: *Corruption increases the optimal level of import tariff and reduces the optimal level of pollution tax.*

The second order conditions appear in Appendix D.

2.6.1 Best response functions

In like manner, we compute slopes of the best response functions when policy outcomes are not at the first-best levels. These are obtained by total differentiation of the first-order conditions associated with the political equilibrium (condition (126)).

Using results from Section 2.5.1-Section 2.5.5 and (119) condition (126) can be expanded as

$$W^P_t = \left(\frac{X}{n}p_Y Y_t\right) + a\left[\Big(-(Xp_t + Yq_t)\Big) + \left(\frac{X}{n}p_Y Y_t\right) + \Big(\tau X_t\Big) + \Big(-n\theta X_t\Big) + \Big(Y + tY_t\Big)\right] = 0; \tag{139}$$

for $\chi = t$, i.e., home import tariff.

Total differentiation of (143) and further substitution of (D8) and (D11) yield

$$W^P_{tt}dt + W^P_{t\tau}d\tau + W^P_{t\tau^*}d\tau^* = 0; \tag{140}$$

where

$$W^P_{t\tau^*} = \frac{1}{n}p_Y Y_t X^*_\tau + a\left\{-(p_t X_{\tau^*} + q_t Y_{\tau^*}) + \frac{1}{n}p_Y Y_t X_{\tau^*} + Y_{\tau^*}\right\} = -\frac{1}{n^3D^2}2p_X p_Y^2 + a\left(\frac{1}{n^3D^2}p_X(4p_X^2 - p_Y^2)\right). \tag{141}$$

Re-arranging terms in (140) and using the results from (29) and (32)-(39) give us

$$dt = \lambda^P_1 d\tau + \lambda^P_2 d\tau^*; \tag{142}$$

where,

$$\lambda_1^P = \frac{W^P_{t\tau}}{(-W^P_{tt})} = \frac{\overbrace{\frac{1}{n^3D^2}4p_X^2p_Y}^{(\text{-})} + \overbrace{a\Big(-\frac{1}{n^3D^2}p_Y(4p_X^2 - p_Y^2)\Big)}^{(+)}}{(-W^P_{tt})}; \tag{143}$$

and

$$\lambda_2^P = \frac{W^P_{t\tau^*}}{(-W^P_{tt})} = -\frac{\overbrace{-\frac{1}{n^3D^2}2p_Xp_Y^2}^{(+)} + \overbrace{a\Big(\frac{1}{n^3D^2}p_X(4p_X^2 - p_Y^2)\Big)}^{(\text{-})}}{(-W^P_{tt})}. \tag{144}$$

Comparing (143) with (106) and (144) with (107), it is evident that the signs of coefficients in the differential expression in (142) are ambiguous. This ambiguity arises on account of the political aspirations of the policy maker. While the denominators in (143) and (144) are required to be negative, the signs of λ_1^P and λ_2^P are determined by the relative strengths of the two terms in the numerator. The first term in the numerators of (143) and (144) refers to the impact of campaign contributions accruing to the policy maker on account of policy interdependencies, with a negative and positive sign, respectively. The negative sign in (143) implies that a rise in home pollution tax reduces the output and profits of the home firm, which translates into lower campaign contribution and the resultant lower marginal benefit from tariff accruing to the policy maker. This causes home import tariff to fall in response to a rise in home pollution tax, as discussed in the study by Mehra (2010). In contrast, a rise in foreign pollution tax increases home output and profits which results in higher campaign funds and thereby, raises the marginal benefit from import tariff. This induces the policy maker to raise home import tariff. The second term in the numerators of (143) and (144) replicates the effects discussed in Proposition 9 under the social optimum. Furthermore, the direction of change of home tariff in response to a change in home as well as foreign pollution tax depends on the relative weight that the policy maker attaches to campaign funds. Mathematically, we have

$$\lim_{a\to\infty} \lambda_1^P = \lambda_1^G > 0; \tag{145}$$

and

$$\lim_{a\to\infty} \lambda_2^P = \lambda_2^G < 0. \tag{146}$$

In case the policy maker places a greater relative weight on campaign funds, i.e., 'a' takes a smaller value, considerations for campaign funds rise relative to that of aggregate social welfare. Consequently, the politically motivated government responds to a higher pollution tax at home with a lower domestic protection. Higher pollution taxes increase the cost of production incurred by the firm which translates into lower campaign funding. The policy maker, being more corrupt, has less incentive to raise domestic protection. When foreign pollution tax rises, the home government raises home tariff. Hence, policy interdependencies work in the opposite direction of those evaluated under the social optimum when the government is highly corrupt. As the level of corruption falls (i.e., 'a' rises), the coefficients λ_1^P and λ_2^P converge to their corresponding 'socially optimal' levels λ_1^P and λ_2^P. For non-extreme values of 'a', the signs of λ_1^P and λ_2^P are ambiguous.

Proposition 15: *The direction of movement of home (foreign) tariff in response to a rise in both home (foreign) pollution tax and foreign (home) pollution tax is ambiguous.*

(i) When the politically motivated government is highly corrupt, the direction of movement reverses in comparison to that of the social optimum.

(ii) As the government tends to place more weight on aggregate social welfare as opposed to contributions from the lobby, the direction of movement tends to coincide with that of the social optimum.

(iii) Again, foreign (home) tariff has no first-order effect on home (foreign) tariff.

Turning towards the best response function of home pollution tax, we utilise the results obtained in Section 2.5.1-Section 2.5.5, and the condition in (119) to express condition (126) as

$$\begin{aligned} W^P_\tau = \Big(\frac{X}{n}p_YY_\tau + \frac{X^*}{n^*}p^*{}_{Y^*}Y^*_\tau - \widetilde{X}\Big) + a\Big[\Big(-(Xp_\tau + Yq_\tau)\Big) + \Big(\frac{X}{n}p_YY_\tau + \frac{X^*}{n^*}p^*{}_{Y^*}Y^*_\tau - \widetilde{X}\Big) \\ + \Big(\widetilde{X} + \tau\widetilde{X_\tau}\Big) + \Big(-n\theta\widetilde{X_\tau}\Big) + \Big(tY_\tau\Big)\Big] = 0; \end{aligned} \tag{147}$$

for $\chi = \tau$, i.e., home pollution tax.

Total differentiation of (147) and further substitution of expressions (D9) and (D12) yield

$$W^P_{\tau\tau}d\tau + W^P_{\tau t}dt + W^P_{\tau t^*}dt^* + W^P_{\tau\tau^*}d\tau^* = 0. \tag{148}$$

where

$$W^P_{\tau t^*} = \left(\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{t^*} - X^*_{t^*}\right) + a\left\{\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{t^*}\right\} = -\frac{1}{n^{*3}D^{*2}}8p^{*3}_{X^*} + a\left(-\frac{1}{n^{*3}D^{*2}}2p^*_{X^*}p^{*2}_{Y^*}\right); \tag{149}$$

and

$$\begin{aligned} W^P_{\tau\tau^*} = &\left(\frac{1}{n}p_Y Y_\tau X_{\tau^*} + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{\tau^*} - X_{\tau^*} - X^*_{\tau^*}\right) + a\left\{-(p_\tau X^*_\tau + q_\tau Y^*_\tau) + \frac{1}{n}p_Y Y_\tau X^*_\tau \right. \\ &\left. + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{\tau^*}\right\} = 4\left(\frac{1}{n^3D^2}p^2_X p_Y + \frac{1}{n^{*3}D^{*2}}p^{*2}_{X^*}p^*_{Y^*}\right) + a\left(\frac{1}{n^{*3}D^{*2}}p*^3_{Y^*}\right). \end{aligned} \tag{150}$$

Re-arranging terms in (148) and using results in (29) and (32)-(39) we get

$$d\tau = \Lambda^P_1 dt + \Lambda^P_2 dt^* + \Lambda^P_3 d\tau^*; \tag{151}$$

where,

$$\Lambda^P_1 = \frac{W^P_{\tau t}}{(-W^P_{\tau\tau})} = \frac{\overbrace{\frac{1}{n^3D^2}4p^2_X p_Y}^{(-)} + \overbrace{a\left(-\frac{1}{n^3D^2}p_Y(4p^2_X - p^2_Y)\right)}^{(+)}}{(-W^P_{\tau\tau})}; \tag{152}$$

$$\Lambda^P_2 = \frac{W^P_{\tau t^*}}{(-W^P_{\tau\tau})} = \frac{\overbrace{-\frac{1}{n^{*3}D^{*2}}8p^{*3}_{X^*}}^{(+)} + \overbrace{a\left(-\frac{1}{n^{*3}D^{*2}}2p^*_{X^*}p^{*2}_{Y^*}\right)}^{(+)}}{(-W^P_{\tau\tau})} > 0; \tag{153}$$

and

$$\Lambda^P_3 = \frac{W^P_{\tau\tau^*}}{(-W^P_{\tau\tau})} = \frac{\overbrace{4\left(\frac{1}{n^3D^2}p^2_X p_Y + \frac{1}{n^{*3}D^{*2}}p^{*2}_{X^*}p^*_{Y^*}\right)}^{(-)} + \overbrace{a\left(\frac{1}{n^{*3}D^{*2}}p*^3_{Y^*}\right)}^{(-)}}{(-W^P_{\tau\tau})} < 0. \tag{154}$$

The coefficients of the differential expression in (151) consist of the additional effect of campaign contributions on the interdependencies in policies, which is given by the first term in the numerator for each coefficient. From the second-order conditions, the denominator is required to be negative. Therefore, apart from the sign of Λ^P_1, which is indeterminate, the signs of the remaining coefficients Λ^P_2 and Λ^P_3 are unambiguously positive and negative, respectively. Considerations for campaign funds work in the opposite direction of that of the aggregate social welfare. When home tariff rises, the resultant rise in output and profit of the home firm amplifies the lobbying effort. This reduces home pollution tax owing to an increase in the marginal benefit from reduction in pollution tax at home. In contrast, considerations for aggregate social welfare requires tightening of the environmental policy at home (given by the second term in the numerator). Therefore, our result stands in sharp contrast to the one derived in Mehra (2010) where a higher tariff at home always induces the government to tighten the environmental regulation. When foreign protection rises, the home firm suffers competitive disadvantage which is reflected in lower campaign contributions. The politically motivated government at home, as a result, tightens the environmental policy at home, thereby harming the lobby. This effect works in the same direction as that of the second term in the numerator in (153). The first term in the numerator of (154) shows that a tougher environmental policy in the foreign country works in favour of the home lobby in terms of a cost advantage resulting in an increase in lobbying effort and hence a reduction in home pollution tax. Considerations for aggregate social welfare also causes home pollution to fall in response to a rise in foreign pollution tax.

Also, it must be noted that the coefficients of (151) converge to the corresponding values in (112) when the policy maker places less relative weight to campaign funds, i.e.,

$$\lim_{a\to\infty} \Lambda_1^P = \Lambda_1^G > 0; \tag{155}$$

$$\lim_{a\to\infty} \Lambda_2^P = \Lambda_2^G > 0; \tag{156}$$

$$\lim_{a\to\infty} \Lambda_3^P = \Lambda_3^G < 0. \tag{157}$$

Proposition 16: *The response to a rise in home (foreign) import tariff on home (foreign) pollution tax is ambiguous. While the political consideration associated with this change tends to lower home (foreign) pollution tax, considerations for aggregate social welfare lead to the tightening of environmental regulations at home (foreign).*

Furthermore, home (foreign) pollution tax increases when:

(i) the foreign (home) country raises its import tariff;

(ii) the foreign (home) country reduces pollution tax.

The response of t with respect to a change in t^* is given by

$$\left.\frac{dt}{dt^*}\right|_P = \left.\frac{dt}{d\tau}\right|_P . \left.\frac{d\tau}{dt^*}\right|_P = \lambda_1^P.\Lambda_2^P \tag{158}$$

Expression (158) shows that the change in foreign tariff generates a second order effect on home tariff via the change in home pollution tax. Owing to the ambiguity of the sign of λ_1^P, the direction of movement of tariffs is indeterminate.

Having characterised the social and political equilibrium, we now consider specific forms of the demand functions and examine the signs of comparative static effects of the policy variables under a host of parametric configurations. The specific functional forms help us to derive explicit solutions as well as the direction of movement of one of the policy variables in response to a change in any other policy variable (holding other policies as given) under the political equilibrium.

2.6.2 A case of Quasilinear Quadratic Utility (QQU)

The Quasilinear Quadratic Utility Model (QQUM), widely popular in the oligopoly literature, ensures closed-form solutions and is characterised by linear demand functions for differentiated products. The utility function in expression (1), thereby, takes the following form:

$$U = z + (x + y) - \frac{1}{2}(x^2 + 2\gamma xy + y^2) - \theta(X + X^*); \tag{159}$$

with

$$u(x, y) = (x + y) - \frac{1}{2}(x^2 + 2\gamma xy + y^2);$$

and

$$\gamma^2 < 1 \;\&\; \gamma > 0 \implies \gamma \in (0, 1);$$

where γ denotes the degree of product differentiation. γ being strictly positive implies that the products are substitutes and as this value rises, the degree of differentiation falls. Utility maximisation yields linear inverse demands

$$p = 1 - x - \gamma y; \tag{160}$$

and

$$q = 1 - y - \gamma x; \tag{161}$$

Similarly, demand functions for the foreign country are symmetrically given by

$$p^* = 1 - x^* - \gamma^* y^*; \tag{160'}$$

and

$$q^* = 1 - y^* - \gamma^* x^*; \tag{161'}$$

where γ^* denotes the degree of product differentiation in the foreign country. We now derive the regularity conditions for unique and stable equilibrium, assuming that the degree of product differentiation being identical across the border ($\gamma = \gamma^*$) along with the population in both home and the foreign country being normalised to unity (i.e., $n = n^* = 1$).[6] We restrict our analysis to the reaction functions of the home country. Assuming symmetry, the results can be derived analogously for the foreign country. In case of demand functions (160), (161), (160') and (161'), the second-order conditions in (D4) become

$$W_{tt}^P = \frac{2\gamma^2 - 3a(4-\gamma^2)}{(4-\gamma^2)^2} < 0; \tag{162}$$

$$W_{\tau\tau}^P = \frac{16 - a(12 - 5\gamma^2)}{(4-\gamma^2)^2} < 0; \tag{163}$$

$$W_{\tau t}^P = W_{t\tau}^P = \frac{-4\gamma + a\gamma(4-\gamma^2)}{(4-\gamma^2)^2}. \tag{164}$$

From the expressions (162) and (163), the following sufficient conditions are obtained, respectively:

$$a > \frac{2\gamma^2}{3(4-\gamma^2)} = a' \text{ (say)}; \tag{R1}$$

$$a > \frac{16}{12 - 5\gamma^2} = a'' \text{ (say)}. \tag{R2}$$

In order to ensure negative definiteness of the Hessian matrix H^P, the following condition must be met:

$$\sigma_1 a^2 + \sigma_2 a + \sigma_3 > 0 \iff (a - a_1)(a - a_2) > 0; \tag{R3}$$

where

$$\sigma_1 = -\gamma^6 + 23\gamma^4 - 112\gamma^2 + 144;$$
$$\sigma_2 = 2\gamma^4 + 56\gamma^2 - 192;$$
$$\sigma_3 = 16\gamma^2;$$

and

$$a_1 = \frac{-\sigma_2 - \sqrt{\sigma_2^2 - 4\sigma_1\sigma_3}}{2\sigma_1}; \; a_2 = \frac{-\sigma_2 + \sqrt{\sigma_2^2 - 4\sigma_1\sigma_3}}{2\sigma_1};$$

are the roots of the quadratic equation in a, i.e.,

$$\sigma_1 a^2 + \sigma_2 a + \sigma_3 = 0 \iff (a - a_1)(a - a_2) = 0;$$

with $0 < a_1 < a_2$, given that $a > 0$.

Now, condition (R3) depicts that, either

$$a > a_1 \text{ and } a > a_2 \implies a \in (a_2, \infty); \tag{R3.1}$$

or

$$a < a_1 \text{ and } a < a_2 \implies a \in (0, a_1). \tag{R3.2}$$

From conditions (R3.1) and (R3.2), we get

$$a \in (0, a_1) \cup (a_2, \infty). \tag{R3'}$$

The sufficient condition in (R3') derived from conditions (R3.1) and (R3.2) imply that the relative weight attached by the policy maker to aggregate social welfare should take a specific range of values in order to ensure stability and uniqueness of equilibrium. In case the the weight, a takes values within the interval $[a_1, a_2]$, the sufficient conditions for local maximum of the welfare function of the politically motivated regulator are not met.

[6] An important point to note here is that alternative scenarios where the trading nations are asymmetric in terms of the degree of product differentiation (γ and γ^*), the weight attached to aggregate social welfare relative to political rents by the incumbent government (a and a^*) or the population size (n and n^*) cannot be completely ruled out. However, the direction of movement of trade and environmental policies are quite likely to be unaltered. Therefore, our results hold qualitatively across both the scenarios. The only change would be in terms of the magnitude of the coefficients in the differentials in (142) and (151). Consequently, the equilibrium outcomes would differ from those obtained under symmetry.

Hereafter, with focus on the home country, we can compute numerical values of a for any given value of γ. Table 3 compiles the results.

Table 3: Regulatory conditions

γ	a'	a''	a_1	a_2	Value of a satisfying (R1), (R2) and (R3')
$0^\dagger$	0	1.33333	0	1.33333	$a \in (1.33333, \infty)$
0.1	0.00167	1.33891	0.00084	1.33901	$a \in (0, 0.00084) \cup (1.33901, \infty)$
0.2	0.00673	1.35593	0.00338	1.35633	$a \in (0, 0.00338) \cup (1.35633, \infty)$
0.3	0.01535	1.38528	0.00775	1.38626	$a \in (0, 0.00775) \cup (1.38626, \infty)$
0.4	0.02778	1.42857	0.01413	1.43054	$a \in (0, 0.01413) \cup (1.43054, \infty)$
0.5	0.04444	1.48837	0.02283	1.49201	$a \in (0, 0.02283) \cup (1.49201, \infty)$
0.6	0.06593	1.56863	0.03430	1.57506	$a \in (0, 0.03430) \cup (1.57506, \infty)$
0.7	0.09307	1.67539	0.04917	1.68667	$a \in (0, 0.04917) \cup (1.68667, \infty)$
0.8	0.12698	1.81818	0.06837	1.83815	$a \in (0, 0.06837) \cup (1.83815, \infty)$
0.9	0.16928	2.01258	0.09324	2.04901	$a \in (0, 0.09324) \cup (2.04901, \infty)$
$1.0^\ddagger$	0.22222	2.28571	0.12579	2.35570	$a \in (0, 0.12579) \cup (2.35570, \infty)$

Note: † and ‡ are extreme cases where the two varieties of the traded good are independent and identical, respectively. The non-extreme values of the parameter γ indicate that the two varieties are differentiated. Higher values of γ imply a lower degree of product differentiation.

From the table, we can deduce the following:

$$a' < a'' < a_2 \; \forall \; \gamma. \tag{R4}$$

Therefore,

$$a > a'' \implies a > a'; \; [(\text{R2}) \text{ is sufficient for } (\text{R1})]. \tag{R5}$$

and

$$a > a_2 \implies a > a''; \; [(\text{R3.1}) \text{ is sufficient for } (\text{R2})]. \tag{R6}$$

Hence, the regularity condition is given by (R3'), depicted in the last column of Table 3.

Now, we turn towards a simulation exercise by assigning different combinations of parameter values to understand the policy interdependencies where our results turned out to be ambiguous, i.e., the signs of the numerators of λ_1^P, λ_2^P and Λ_1^P in expressions (143), (144) and (152), respectively. Utilising the demand functions from (160), (161), (160') and (161'), we have

$$W_{t\tau}^P = W_{\tau t}^P = \frac{-4\gamma + a\gamma(4-\gamma^2)}{(4-\gamma^2)^2}; \tag{S1}$$

and

$$W_{t\tau^*}^P = \frac{2\gamma^2 - a(4-\gamma^2)}{(4-\gamma^2)^2}. \tag{S2}$$

The signs of both the expressions in (S1) and (S2), which depend on parameters a and γ are ambiguous. These refer to the numerators of the coefficients in (143), (144) and (152), respectively, which determine their sign. The simulation exercise helps us in understanding the policy interdependencies under alternative parametric specifications. Table 4 compiles the results.

Table 4: Policy interdependencies

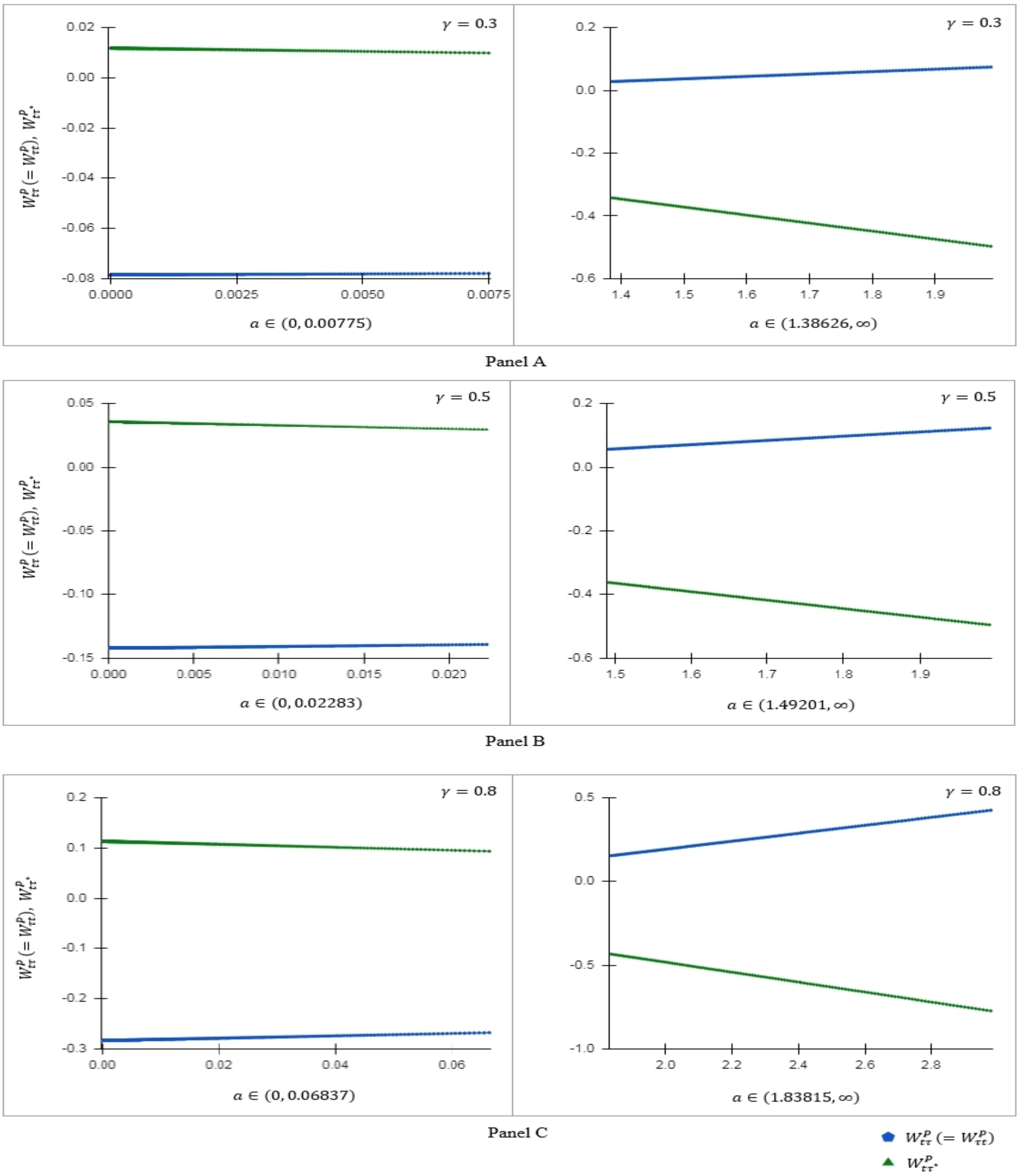
Parameter	γ	a	$W^P_{t\tau}(=W^P_{\tau t})$	$W^P_{t\tau^*}$
Run 1	0	1.400	0	-0.350
Run 2		2.000	0	-0.500
Run 3		3.000	0	-0.750
Run 4		5.200	0	-1.300
Run 5	0.3	0.001	-0.078	0.012
Run 6		0.006	-0.078	0.010
Run 7		1.400	0.029	-0.346
Run 8		2.000	0.075	-0.500
Run 9	0.5	0.010	-0.141	0.033
Run 10		0.020	-0.140	0.030
Run 11		1.500	0.058	-0.364
Run 12		2.000	0.124	-0.500
Run 13	0.8	0.010	-0.281	0.110
Run 14		0.040	-0.274	0.101
Run 15		1.900	0.168	-0.452
Run 16		2.000	0.193	-0.482
Run 17	1.0	0.010	-0.441	0.219
Run 18		0.090	-0.414	0.192
Run 19		2.500	0.389	-0.611
Run 20		3.000	0.556	-0.778

Numerical simulations for $\gamma =$ 0.3, 0.5, 0.8, 1; each with four runs under the regulatory conditions show that $W^P_{t\tau}$ $(= W^P_{\tau t})$ is negative when (R3.2) holds and positive when the sufficient condition (R3.1) is met. The converse is true for $W^P_{t\tau^*}$. Since $W^P_{t\tau}$ proportionately varies with γ, it takes the value 0 for $\gamma = 0$. Furthermore, condition (R3.1) ensures that $W^P_{t\tau^*}$ is negative.

Therefore, Proposition 15 and Proposition 16 are verified for a large range of values of the relative weight attached to aggregate social welfare by the policy maker as compiled in the last column of Table 3. Lower values of a indicate that the policy maker is highly corrupt and gives in to campaign contributions. Accordingly, policy responses work in the opposite direction than the ones that maximise aggregate social welfare. We find that considerations for campaign funds outweigh welfare goals when condition (R3.2) is met. Consequently, the numerators of λ_1^P, Λ_1^P become negative and that of λ_2^P turns out to be positive. Political considerations tend to lower import tariff in response to a rise pollution tax in both the home and the foreign country. In a similar fashion, a rise in import tariff raise pollution tax in both the home and foreign country. A higher pollution tax at home drives down profits of the home firm and the resultant reduction in campaign funds lowers domestic protection. On the contrary, higher domestic protection increases campaign contributions by the home firm on account of higher firm profits which results in a laxer environmental regulation. When foreign pollution tax rises, home output and profits rise resulting in higher domestic protection.

Proposition 17: *The response to a rise in home (foreign) import tariff (pollution tax) on home (foreign) pollution tax (import tariff) is negative when condition (R3.2) is met. Furthermore, condition (R3.2) also ensures that home (foreign) import tariff is increasing in foreign (home) pollution tax.*

Also, the response of t with respect to a change in t^* is negative for $\lambda_1^P < 0$.



Source: Authors' calculations from a numerical simulation exercise

Figure 3: Effects of a change in the relative weight on aggregate social welfare, a on $W^P_{t\tau}(=W^P_{\tau t})$ and $W^P_{t\tau *}$

Figure 3 illustrates the results of Proposition 17 for $\gamma = 0.3,\ 0.5,\ 0.8$ in Panel (A), (B) and (C), respectively. Utilising the regulatory conditions (from Table 3), several iterations suggest that the absolute value of $W^P_{t\tau}(=W^P_{\tau t})$ rises as the policy maker attaches a higher relative weight on aggregate social welfare, whereas it falls in case of the term $W^P_{t\tau *}$. This indicates a gradual decline in the marginal benefit from campaign contributions.

Therefore, political considerations of the regulator, i.e., the relative weight that the regulator attaches to aggregate social welfare is a significant determinant of the direction of movement of any pair of policies. Compared to the baseline scenario of social optimum, the best response of home import tariff to a change in home pollution tax as well as foreign pollution tax is altered under the political equilibrium when the regulator becomes more corrupt. This phenomenon also aligns with the best response of home pollution tax to a change in home import tariff. In case of a change in foreign import tariff and foreign pollution tax, the relationship remains unaltered. The relationship between home import tariff and foreign import tariff is a second-order effect and is determined by the relationship between home pollution tax and home import tariff.

3 Concluding remarks

The present essay provides a theoretical exposition of the political economy of interactions between trade and environmental policies across two trading nations. The study is a novel attempt to capture interdependencies between each of the policies set at home and in the foreign country in a duopoly market setting with trade in differentiated products. Two alternative cases have been considered, namely, the baseline case or the social optimum and the political equilibrium.

The social optimum assumes that the government is benevolent and aims to maximise aggregate social welfare. It is found that free trade is not always an optimal policy choice owing to the duopoly behaviour of firms. Further, the assumption of large open economies entails a consideration for terms-of-trade effects of bilateral trade. In a similar fashion, equilibrium pollution tax may not always reflect true marginal damage. The characterisation of trade and environmental policy interdependencies reveal that both home (foreign) import tariff and pollution tax respond positively to a rise in home (foreign) pollution tax and import tariff. The effect of a rise in foreign (home) import tariff also leads to a rise in home (foreign) pollution tax. However, home (foreign) import tariff responds inversely to an increase in foreign (home) pollution tax. Finally, home (foreign) import tariff rises in response to a rise in foreign (home) import tariff. It is notable that this emanates from a second-order effect, since market segmentation generates a zero first-order effect. Also, environmental policies at home and foreign are inversely related.

The political equilibrium considers the existence of political action groups as well as a politically motivated government who values campaign contributions. Equilibrium import tariffs are higher than the socially optimal level as in Grossman and Helpman (1995) and Mehra (2010). The political inclination induces the government to set a pollution tax which is lower than the socially optimal level as discussed by Damania et al. (2003). Clearly, these wedges taper off as the government becomes less corrupt or when the degree of product differentiation rises. Now, policy interdependencies are affected by the government's considerations for campaign funds. The direction of movement of home (foreign) tariff in response to a rise in both home (foreign) pollution tax and foreign (home) pollution tax is ambiguous. When the politically motivated government is highly corrupt, the direction of movement reverses in comparison to that of the social optimum. Similarly, the response to a rise in home (foreign) import tariff on home (foreign) pollution tax is less predictable. While the political consideration tends to lower home (foreign) pollution tax, considerations for aggregate social welfare leads to the tightening of environmental regulations at home (foreign). Finally, home (foreign) pollution tax increases when: the foreign (home) country raises its import tariff; the foreign (home) country reduces pollution tax. The numerical simulation exercise confirms these results.

Under some regulatory conditions derived for alternative values of the degree of product differentiation, we trace out the direction of movement of policy variables under the political equilibrium. For example, when the regulator is more corrupt, i.e., a lower value of a, say, $a = \{0.001, 0.006\}$ for $\gamma = 0.3$ (runs 5 and 6), the numerator of $\lambda_1^P (= \Lambda_1^P)$ takes negative values, -0.078 and -0.078, respectively. This implies that a rise in home pollution tax (home import tariff) leads to reduction in home import tariff (home pollution tax). Again, the numerator of λ_2^P takes positive values, 0.012 and 0.010, respectively. Therefore, home import tariff rises in response to a rise in foreign pollution tax. These results are in sharp contrast to the ones derived under the social optimum. As the regulator becomes less corrupt, the direction of movement of policies are consistent with that of the social optimum. Here, a takes a higher value, say $a = \{1.900, 2.000\}$ for $\gamma = 0.8$ (runs 15 and 16) the numerator of $\lambda_1^P (= \Lambda_1^P)$ takes positive values, 0.168 and 0.193, respectively. Further, the numerator of λ_2^P takes negative values, -0.452 and -0.482, respectively.

To sum up, our analysis makes a novel attempt to explain policy interdependencies in a two-country, two-Cournot-firm and four-policy setting. The assumption of duopoly markets with trade in differentiated products in a political economy framework provides some useful insights.

The theoretical framework can be extended further by considering price competition between the duopoly firms [e.g. Kagitani (2009)]. Also, the implications are likely to differ when firms make sequential moves. It would also be interesting to see how the results alter when consumers derive utility from consumption of more than two varieties of the traded good. Furthermore, our numerical simulation exercise is based on the assumption that the trading nations are symmetric in terms of the parameters capturing degrees of product differentiation and honesty of the incumbent politician as well as population size, which is highly unlikely to hold in reality. While our results hold qualitatively irrespective of how these parameters compare across the nations, the equilibrium outcomes in terms of import tariffs and pollution taxes are likely to be affected due to asymmetries in the above parameters. The scope our research being confined to examining movements and interdependencies between the two policies, comparisons between the equilibrium outcomes on account of these asymmetries would potentially be a compelling area to be explored.

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Appendix A

The second-order conditions for profit maximisation are given as

$$\pi_{XX} \equiv \frac{\partial(\pi_X)}{\partial X} = \frac{X}{n^2}\frac{\partial}{\partial X}\left(\frac{\partial p}{\partial X}\right) + \frac{2}{n}\frac{\partial p}{\partial X} < 0; \tag{A1}$$

$$\pi_{XY} \equiv \frac{\partial(\pi_X)}{\partial Y} = \frac{X}{n^2}\frac{\partial}{\partial Y}\left(\frac{\partial p}{\partial X}\right) + \frac{1}{n}\frac{\partial p}{\partial Y} < 0; \tag{A2}$$

$$\pi_{X^*X^*} \equiv \frac{\partial(\pi_{X^*})}{\partial X^*} = \frac{X^*}{n^{*2}}\frac{\partial}{\partial X^*}\left(\frac{\partial p^*}{\partial X^*}\right) + \frac{2}{n^*}\frac{\partial p^*}{\partial X^*} < 0; \tag{A3}$$

$$\pi_{X^*Y^*} \equiv \frac{\partial(\pi_{X^*})}{\partial Y^*} = \frac{X^*}{n^{*2}}\frac{\partial}{\partial Y^*}\left(\frac{\partial p^*}{\partial X^*}\right) + \frac{1}{n^*}\frac{\partial p^*}{\partial Y^*} < 0; \tag{A4}$$

$$\pi^*_{YY} \equiv \frac{\partial(\pi^*_Y)}{\partial Y} = \frac{Y}{n^2}\frac{\partial}{\partial Y}\left(\frac{\partial q}{\partial Y}\right) + \frac{2}{n}\frac{\partial q}{\partial Y} < 0; \tag{A5}$$

$$\pi^*_{YX} \equiv \frac{\partial(\pi^*_Y)}{\partial X} = \frac{Y}{n^2}\frac{\partial}{\partial X}\left(\frac{\partial q}{\partial Y}\right) + \frac{1}{n}\frac{\partial q}{\partial X} < 0; \tag{A6}$$

$$\pi^*_{Y^*Y^*} \equiv \frac{\partial(\pi^*_{Y^*})}{\partial Y^*} = \frac{Y^*}{n^{*2}}\frac{\partial}{\partial Y^*}\left(\frac{\partial q^*}{\partial Y^*}\right) + \frac{2}{n^*}\frac{\partial q^*}{\partial Y^*} < 0; \tag{A7}$$

$$\pi^*_{Y^*X^*} \equiv \frac{\partial(\pi^*_{Y^*})}{\partial X^*} = \frac{Y^*}{n^{*2}}\frac{\partial}{\partial X^*}\left(\frac{\partial q^*}{\partial Y^*}\right) + \frac{1}{n^*}\frac{\partial q^*}{\partial X^*} < 0. \tag{A8}$$

Conditions (A1), (A3), (A5) and (A7) show that the slope of the marginal revenue curves of the firm in the home and the foreign country are declining. On the other hand, conditions (A2), (A4), (A6) and (A8) imply that marginal revenues of a firm serving in both the home and the foreign market decline with a rise in the output level of its rival.

Recall that the sign of mixed partials are negative if the choice variables are quantities. In addition to this, the own price effect is greater than the cross price effect. Hence, the stability conditions can be stated as

$$D = \pi_{XX}\pi^*_{YY} - \pi_{XY}\pi^*_{YX} > 0;$$

and

$$D^* = \pi_{X^*X^*}\pi^*_{Y^*Y^*} - \pi_{X^*Y^*}\pi^*_{Y^*X^*} > 0.$$

Table 5: Second-order derivatives under under the linearity assumption

Second-order derivative	Value	Second-order derivative	Value
π_{XX}	$\frac{1}{n}2p_X$	$\pi^*{}_{YY}$	$\frac{1}{n}2q_Y$
π_{XY}	$\frac{1}{n}p_Y$	$\pi^*{}_{YX}$	$\frac{1}{n}2q_X$
$\pi_{X^*X^*}$	$\frac{1}{n^*}2p^*_{X^*}$	$\pi^*_{Y^*Y^*}$	$\frac{1}{n^*}2q^*_{Y^*}$
$\pi_{X^*Y^*}$	$\frac{1}{n^*}2p^*_{Y^*}$	$\pi^*_{Y^*X^*}$	$\frac{1}{n^*}2q^*_{X^*}$
D	$\frac{1}{n^2}(4p^2_X - p^2_Y)$	D^*	$\frac{1}{n^{*2}}(4p^{*2}_{X^*} - p^{*2}_{Y^*})$

Appendix B

Comparative statics

We compute comparative static effects of change of policy instruments on outputs. From the first-order conditions (12) and (18), we have

$$\pi_X \equiv \pi_X(X, Y, \tau) = 0;$$

$$\pi_Y^* \equiv \pi_Y^*(X, Y, \tau^*, t) = 0.$$

Total differentiation of the above equations yields

$$\pi_{XX}dX + \pi_{XY}dY = -\pi_{X\tau}d\tau;$$

$$\pi_{YX}^*dX + \pi_{YY}^*dY = -\pi_{Y\tau^*}^*d\tau^* - \pi_{Yt}^*dt.$$

In matrix notation [using results from (13) and (19)], we have

$$\begin{pmatrix} \pi_{XX} & \pi_{XY} \\ \pi_{YX}^* & \pi_{YY}^* \end{pmatrix} \begin{pmatrix} dX \\ dY \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} d\tau \\ d\tau^* \\ dt \\ dt^* \end{pmatrix};$$

or,

$$D\Omega = A\Gamma;$$

where

$$D = \begin{pmatrix} \pi_{XX} & \pi_{XY} \\ \pi_{YX}^* & \pi_{YY}^* \end{pmatrix};\ \Omega = \begin{pmatrix} dX \\ dY \end{pmatrix};\ A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix};\ \text{and}\ \Gamma = \begin{pmatrix} d\tau \\ d\tau^* \\ dt \\ dt^* \end{pmatrix}.$$

Therefore,

$$\Omega = D^{-1}A\Gamma;$$

where

$$D^{-1} = \frac{1}{D}(Adjoint\ D)^T = \frac{1}{D}\begin{pmatrix} \pi_{YY}^* & -\pi_{XY} \\ -\pi_{YX}^* & \pi_{XX} \end{pmatrix};$$

$$\begin{pmatrix} dX \\ dY \end{pmatrix} = \frac{1}{D}\begin{pmatrix} \pi_{YY}^* & -\pi_{XY} \\ -\pi_{YX}^* & \pi_{XX} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} d\tau \\ d\tau^* \\ dt \\ dt^* \end{pmatrix}.$$

Similarly, the first-order conditions (15) and (21) can be used to find the following equation

$$\begin{pmatrix} dX^* \\ dY^* \end{pmatrix} = \frac{1}{D^*}\begin{pmatrix} \pi_{Y^*Y^*}^* & -\pi_{X^*Y^*} \\ -\pi_{Y^*X^*}^* & \pi_{X^*X^*} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} d\tau \\ d\tau^* \\ dt \\ dt^* \end{pmatrix}.$$

The above equations are used to derive equations (32)-(39). Table 6 compiles the effects using results from Table 5.

Table 6: Comparative statics

Comparative statics	Value	Comparative statics	Value
X_t	$-\frac{1}{nD}p_Y$	Y_t	$\frac{1}{nD}2p_X$
X_{t^*}	0	Y_{t^*}	0
X_t^*	0	Y_t^*	0
$X_{t^*}^*$	$\frac{1}{n^*D^*}2p_{X^*}^*$	$Y_{t^*}^*$	$-\frac{1}{n^*D^*}2p_{Y^*}^*$
X_τ	$\frac{1}{nD}p_X$	Y_τ	$-\frac{1}{nD}p_Y$
X_{τ^*}	$-\frac{1}{nD}p_Y$	Y_{τ^*}	$\frac{1}{nD}p_X$
X_τ^*	$\frac{1}{n^*D^*}p_{X^*}^*$	Y_τ^*	$-\frac{1}{n^*D^*}p_{Y^*}^*$
$X_{\tau^*}^*$	$-\frac{1}{n^*D^*}p_{Y^*}^*$	$Y_{\tau^*}^*$	$\frac{1}{n^*D^*}p_{X^*}^*$

Effect on consumer surplus

Considering the results from (32)-(39) and simplifying further by substituting the results from (29) [see Table 6], we get

$$p_t = \frac{1}{n^2D}p_Xp_Y > 0; \tag{B1}$$

$$q_t = \frac{1}{n^2D}\left(2p_X^2 - p_Y^2\right) > 0; \tag{B2}$$

$$p_\tau = \frac{1}{n^2D}\left(2p_X^2 - p_Y^2\right) > 0; \tag{B3}$$

and

$$q_\tau = \frac{1}{n^2D}p_Xp_Y > 0; \tag{B4}$$

Given the sign restrictions and the assumption that own price effects exceed cross price effects, expressions (B1)-(B4) turn out to be strictly positive. Therefore, a rise in home import tariff or pollution tax would drive up the domestic price of both the output produced for home consumption as well as imports from abroad.

Effect on firm's profits

The first-order conditions of profit maximisation yield $\pi_X = 0$ and $\pi_{X^*} = 0$. Hence,

$$\frac{\partial\pi}{\partial t} = \pi_Y Y_t + \pi_{Y^*}Y_t^* + \pi_t; \tag{B5}$$

$$\frac{\partial\pi}{\partial\tau} = \pi_Y Y_\tau + \pi_{Y^*}Y_\tau^* + \pi_\tau. \tag{B6}$$

Differentiating home firm's profit with respect to t, τ, Y and Y^* yields, respectively

$$\pi_t = 0; \tag{B7}$$

$$\pi_\tau = -\widetilde{X}; \tag{B8}$$

$$\pi_Y = \frac{X}{n}p_Y; \tag{B9}$$

$$\pi_{Y^*} = \frac{X^*}{n^*}p_{Y^*}^*. \tag{B10}$$

Now, using $Y_t^* = 0$ from (34) and the results in (B7)-(B10) in (B5) and (B6), we get

$$\frac{\partial\pi}{\partial t} = \frac{X}{n}p_Y Y_t = \frac{X}{n^2D}2p_Xp_Y > 0; \tag{B11}$$

$$\frac{\partial\pi}{\partial\tau} = \frac{X}{n}p_Y Y_\tau + \frac{X^*}{n^*}p_{Y^*}^*Y_\tau^* - \widetilde{X} = -\left(\frac{X}{n^2D}p_Y^2 + \frac{X^*}{n^{*2}D^*}p^{*2}_{Y^*} + \widetilde{X}\right) < 0. \tag{B12}$$

Appendix C

Non-cooperative equilibrium policy outcomes under the social optimum

Proof:

The non-cooperative equilibrium import tariff and pollution tax for the home country under the baseline case are obtained from the first-order conditions

$$\frac{\partial W^G}{\partial t} = \frac{\partial (nS)}{\partial t} + \frac{\partial \pi}{\partial t} + \frac{\partial (\tau \widetilde{X})}{\partial t} - \frac{\partial (n\theta \widetilde{X})}{\partial t} + \frac{\partial (tY)}{\partial t} = 0; \tag{C1}$$

$$\frac{\partial W^G}{\partial \tau} = \frac{\partial (nS)}{\partial \tau} + \frac{\partial \pi}{\partial \tau} + \frac{\partial (\tau \widetilde{X})}{\partial \tau} - \frac{\partial (n\theta \widetilde{X})}{\partial \tau} + \frac{\partial (tY)}{\partial \tau} = 0. \tag{C2}$$

Derivation of equilibrium import tariff

Using the results from Section 2.5 and Appendix B in (C1), we get

$$-(Xp_t + Yq_t) + \frac{X}{n} p_Y Y_t + \tau X_t - n\theta X_t + Y + tY_t = 0;$$

$$\Rightarrow t = \frac{(Xp_t + Yq_t) - \frac{X}{n} p_Y Y_t - (\tau - n\theta) X_t - Y}{Y_t};$$

$$\Rightarrow t = \frac{Xp_t - \frac{X}{n} p_Y Y_t + Y(q_t - 1) - (\tau - n\theta) X_t}{Y_t};$$

$$\Rightarrow t = \frac{\frac{X}{n}(p_X X_t + p_Y Y_t) - \frac{X}{n} p_Y Y_t + Y(q_t - 1) - (\tau - n\theta) X_t}{Y_t};$$

$$\Rightarrow t^G = \frac{\frac{X}{n} p_X X_t + Y(q_t - 1) - (\tau - n\theta) X_t}{Y_t}; \tag{C3}$$

which yields the expression in (67).

Further substitution of the results from Table 6 in (C3) yields,

$$t^G = \frac{\frac{X}{n} p_X \left(-\frac{1}{nD} p_Y\right) + Y\left\{\frac{1}{n^2 D}(2{p_X}^2 - {p_Y}^2) - 1\right\} - (\tau - n\theta)\left(-\frac{1}{nD} p_Y\right)}{\frac{1}{nD} 2p_X}. \tag{C4}$$

Using the result from expression (30) in (C4), we get

$$t^G = -\frac{X}{n}\frac{p_Y}{2} - \frac{Y}{n} p_X + (\tau - n\theta)\frac{p_Y}{2p_X}; \tag{C5}$$

which is the equilibrium import tariff for the home country.

Derivation of equilibrium pollution tax

We use the results from Section 2.5 and Appendix B in (C2) and get

$$-(Xp_\tau + Yq_\tau) + \frac{X}{n} p_Y Y_\tau + \frac{X^*}{n^*} p^*_{Y^*} Y^*_\tau + \tau \widetilde{X}_\tau - n\theta \widetilde{X}_\tau + tY_\tau = 0;$$

$$\Rightarrow \tau = n\theta + \frac{(Xp_\tau + Yq_\tau) - \frac{X}{n} p_Y Y_\tau - \frac{X^*}{n^*} p^*_{Y^*} Y^*_\tau - tY_\tau}{\widetilde{X}_\tau};$$

$$\Rightarrow \tau = n\theta + \frac{\frac{X}{n}(p_X X_\tau + p_Y Y_\tau) + Yq_\tau - \frac{X}{n} p_Y Y_\tau - \frac{X^*}{n^*} p^*_{Y^*} Y^*_\tau - tY_\tau}{\widetilde{X}_\tau};$$

$$\Rightarrow \tau^G = n\theta + \frac{\frac{X}{n} p_X X_\tau + Yq_\tau - \frac{X^*}{n^*} p^*_{Y^*} Y^*_\tau - tY_\tau}{\widetilde{X}_\tau}; \tag{C6}$$

which gives us the expression in (68).

We further substitute the results from Table 6 in (C6) and get

$$\tau^G = n\theta + \frac{\frac{X}{n} p_X \left(\frac{1}{nD} p_X\right) + Y\left(\frac{1}{n^2 D} p_X p_Y\right) - \frac{X^*}{n^*} p^*_{Y^*}\left(-\frac{1}{n^* D^*} p^*_{Y^*}\right) - t\left(-\frac{1}{nD} p_Y\right)}{\frac{1}{nD} p_X + \frac{1}{n^* D^*} p^*_{X^*}};$$

$$\Rightarrow \tau^G = n\theta + \frac{\frac{X}{n^2 D} p_X^2 + \frac{Y}{n^2 D} p_X p_Y + \frac{X^*}{n^{*2} D^*} {p^*_{Y^*}}^2 - t\left(-\frac{1}{nD} p_Y\right)}{\frac{1}{nD} p_X + \frac{1}{n^* D^*} p^*_{X^*}}; \tag{C7}$$

which is the equilibrium pollution tax for the home country.

Appendix D

Second-order conditions for social optimum

The second-order conditions for the baseline scenario can be written as

$$W_{tt}^G = -(p_t X_t + q_t Y_t) + \frac{1}{n} p_Y Y_t X_t + 2Y_t = \frac{1}{n^3 D^2} 3p_X (4p_X^2 - p_Y^2) < 0; \tag{D1}$$

$$\begin{aligned} W_{\tau\tau}^G &= -(p_\tau X_\tau + q_\tau Y_\tau) + \frac{1}{n} p_Y Y_\tau X_\tau + \frac{1}{n^*} p^*_{Y^*} Y^*_\tau X^*_\tau + \widetilde{X}_\tau \\ &= \frac{1}{n^3 D^2} p_X (4p_X^2 - p_Y^2) + \frac{1}{n^{*3} D^{*2}} 4p^*_{X^*} (2{p^*}^2_{X^*} - {p^*}^2_{Y^*}) < 0. \end{aligned} \tag{D2}$$

The Hessian matrix, H^G associated with the objective function is given by

$$H_G = \begin{pmatrix} W_{tt}^G & W_{t\tau}^G \\ W_{\tau\tau}^G & W_{\tau t}^G \end{pmatrix}; \tag{D3}$$

where

$$W_{tt}^G = \frac{\partial}{\partial t}\left(\frac{\partial W^G}{\partial t}\right);$$

$$W_{t\tau}^G = \frac{\partial}{\partial \tau}\left(\frac{\partial W_G}{\partial t}\right);$$

$$W_{\tau\tau}^G = \frac{\partial}{\partial \tau}\left(\frac{\partial W_G}{\partial \tau}\right);$$

$$W_{\tau t}^G = \frac{\partial}{\partial t}\left(\frac{\partial W_G}{\partial \tau}\right);$$

Now,

$$W_{t\tau}^G = -(p_t X_\tau + q_t Y_\tau) + \frac{1}{n} p_Y Y_t X_\tau + X_t + Y_\tau = -\frac{1}{nD} p_Y > 0 \left(= -\frac{1}{n^3 D^2} p_Y (4{p_X}^2 - {p_Y}^2) > 0 \right); \tag{D4}$$

and

$$W_{\tau t}^G = -(p_\tau X_t + q_\tau Y_t) + \frac{1}{n} p_Y Y_\tau X_t + Y_\tau = -\frac{1}{nD} p_Y > 0 \left(= -\frac{1}{n^3 D^2} p_Y (4{p_X}^2 - {p_Y}^2) > 0 \right). \tag{D5}$$

Also, from (D4) and (D5),

$$W_{t\tau}^G = W_{\tau t}^G. \tag{D6}$$

In order that the second-order conditions for strict local maximum are satisfied, the Hessian matrix must be negative definite, i.e.,

$$|W_{tt}^G| < 0; \ |W_{\tau\tau}^G| < 0; \ |H^G| = \begin{vmatrix} W_{tt}^G & W_{t\tau}^G \\ W_{\tau t}^G & W_{\tau\tau}^G \end{vmatrix} > 0. \tag{D7}$$

Second-order conditions for the political equilibrium

We now check for the sufficient conditions under the political equilibrium. The second-order conditions can be written as

$$\begin{aligned} W_{tt}^P &= \frac{1}{n} p_Y Y_t X_t + a\left[-(p_t X_t + q_t Y_t) + \frac{1}{n} p_Y Y_t X_t + 2Y_t \right] \\ &= -\frac{1}{n^3 D^2} 2p_X p_Y^2 + a\left[\frac{1}{n^3 D^2} 3p_X (4p_X^2 - p_Y^2) \right]; \end{aligned} \tag{D8}$$

$$W_{\tau\tau}^P = \left[\frac{1}{n}p_Y Y_\tau X_\tau + \frac{1}{n^*}p_{Y^*}^* Y_\tau^* X_\tau^* - \widetilde{X}_\tau\right] + a\left[-(p_\tau X_\tau + q_\tau Y_\tau) + \frac{1}{n}p_Y Y_\tau X_\tau + \frac{1}{n^*}p_{Y^*}^* Y_\tau^* X_\tau^* + \widetilde{X}_\tau\right]$$
$$= -8\left[\frac{1}{n^3 D^2}p_X^3 + \frac{1}{{n^*}^3 {D^*}^2}{p^*}_{X^*}^3\right] + a\left[\frac{1}{n^3 D^2}p_X(4p_X^2 - p_Y^2) + \frac{1}{{n^*}^3 {D^*}^2}4p_{X^*}^*(2{p^*}_{X^*}^2 - {p^*}_{Y^*}^2)\right]. \tag{D9}$$

The Hessian matrix, H^P associated with the objective function is given by

$$H_P = \begin{pmatrix} W_{tt}^P & W_{t\tau}^P \\ W_{\tau\tau}^P & W_{\tau t}^P \end{pmatrix}; \tag{D10}$$

where

$$W_{tt}^P = \frac{\partial}{\partial t}\left(\frac{\partial W^P}{\partial t}\right);$$

$$W_{t\tau}^P = \frac{\partial}{\partial \tau}\left(\frac{\partial W_P}{\partial t}\right);$$

$$W_{\tau\tau}^P = \frac{\partial}{\partial \tau}\left(\frac{\partial W_P}{\partial \tau}\right);$$

and

$$W_{\tau t}^P = \frac{\partial}{\partial t}\left(\frac{\partial W_P}{\partial \tau}\right);$$

Now,

$$W_{t\tau}^P = \frac{1}{n}p_Y Y_t X_\tau + a\left[-(p_t X_\tau + q_t Y_\tau) + \frac{1}{n}p_Y Y_t X_\tau + X_t + Y_\tau\right]$$
$$= \frac{1}{n^3 D^2}4p_X^2 p_Y + a\left[-\frac{1}{n^3 D^2}p_Y(4p_X^2 - p_Y^2)\right] \tag{D11}$$

$$W_{\tau t}^P = \left[\frac{1}{n}p_Y Y_\tau X_t - X_t\right] + a\left[-(p_\tau X_t + q_\tau Y_t) + \frac{1}{n}p_Y Y_\tau X_t + X_t + Y_\tau\right]$$
$$= \frac{1}{n^3 D^2}4p_X^2 p_Y + a\left[-\frac{1}{n^3 D^2}p_Y(4p_X^2 - p_Y^2)\right] \tag{D12}$$

Therefore,

$$W_{t\tau}^P = W_{\tau t}^P \tag{D13}$$

In order that the second-order conditions for strict local maximum are satisfied, the Hessian matrix must be negative definite, i.e.,

$$|W_{tt}^P| < 0;\ |W_{\tau\tau}^P| < 0;\ |H^P| = \begin{vmatrix} W_{tt}^P & W_{t\tau}^P \\ W_{\tau t}^P & W_{\tau\tau}^P \end{vmatrix} > 0. \tag{D14}$$

Appendix E

Derivation of slopes of the best response functions under the social optimum

Import tariff at home

Total differentiation of (102) yields

$$-p_t(X_t dt + X_\tau d\tau + X_{\tau^*} d\tau^*) - q_t(Y_t dt + Y_\tau d\tau + Y_{\tau^*} d\tau^*) + \frac{1}{n}p_Y Y_t(X_t dt + X_\tau d\tau + X_{\tau^*} d\tau^*)$$
$$+ X_t d\tau + (Y_t dt + Y_\tau d\tau + Y_{\tau^*} d\tau^*) + Y_t dt = 0. \tag{E1}$$

Collecting coefficients of dt, $d\tau$ and $d\tau^*$ we have

$$\Big\{-(p_tX_t+q_tY_t)+\frac{1}{n}p_YY_tX_t+Y_t+Y_t\Big\}dt+\Big\{-(p_tX_\tau+q_tY_\tau)+\frac{1}{n}p_YY_tX_\tau+X_t+Y_\tau\Big\}d\tau$$
$$+\Big\{-(p_tX_{\tau^*}+q_tY_{\tau^*})+\frac{1}{n}p_YY_tX_{\tau^*}+Y_{\tau^*}\Big\}d\tau^*=0; \tag{E2}$$

Now, (E2) can be expressed as

$$W^G_{tt}dt+W^G_{t\tau}d\tau+W^G_{t\tau^*}d\tau^*=0. \tag{E3}$$

Pollution tax at home

Total differentiation of (108) yields

$$-p_\tau(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)-q_\tau(Y_tdt+Y_\tau d\tau+Y_{\tau^*}d\tau^*)+\frac{1}{n}p_YY_\tau(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)$$
$$+\frac{1}{n^*}p^*{}_{Y^*}Y^*_\tau(X^*_{t^*}dt^*+X^*_\tau d\tau+X^*_{\tau^*}d\tau^*)+\widetilde{X_\tau}d\tau+Y_\tau dt=0. \tag{E4}$$

Collecting coefficients of $d\tau$, dt, dt^* and $d\tau^*$ we have

$$\Big\{-(p_\tau X_\tau+q_\tau Y_\tau)+\frac{1}{n}p_YY_\tau X_\tau+\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_\tau+\widetilde{X_\tau}\Big\}d\tau+\Big\{-(p_\tau X_t+q_\tau Y_t)+\frac{1}{n}p_YY_\tau X_t$$
$$+Y_\tau\Big\}dt+\Big\{\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{t^*}\Big\}dt^*+\Big\{-(p_\tau X^*_\tau+q_\tau Y^*_\tau)+\frac{1}{n}p_YY_\tau X^*_\tau+\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{\tau^*}\Big\}d\tau^*=0; \tag{E5}$$

which can be expressed as

$$W^G_{\tau\tau}d\tau+W^G_{\tau t}dt+W^G_{\tau t^*}dt^*+W^G_{\tau\tau^*}d\tau^*=0. \tag{E6}$$

Derivation of slopes of the best response functions under the political equilibrium

Import tariff at home

Total differentiation of (139) yields

$$\frac{1}{n}p_YY_t(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)+a\Big[-p_t(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)-q_t(Y_tdt+Y_\tau d\tau+Y_{\tau^*}d\tau^*)$$
$$+\frac{1}{n}p_YY_t(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)+X_td\tau+(Y_tdt+Y_\tau d\tau+Y_{\tau^*}d\tau^*)+Y_tdt\Big]=0. \tag{E7}$$

Collecting coefficients of dt, $d\tau$ and $d\tau^*$ we have

$$\Big[\frac{1}{n}p_YY_tX_t+a\Big\{-(p_tX_t+q_tY_t)+\frac{1}{n}p_YY_tX_t+Y_t+Y_t\Big\}\Big]dt+\Big[\frac{1}{n}p_YY_tX_\tau+a\Big\{-(p_tX_\tau+q_tY_\tau)$$
$$+\frac{1}{n}p_YY_tX_\tau+X_t+Y_\tau\Big\}\Big]d\tau+\Big[\frac{1}{n}p_YY_tX^*_\tau+a\Big\{-(p_tX_{\tau^*}+q_tY_{\tau^*})+\frac{1}{n}p_YY_tX_{\tau^*}+Y_{\tau^*}\Big\}\Big]d\tau^*=0; \tag{E8}$$

which can be written as

$$W^P_{tt}dt+W^P_{t\tau}d\tau+W^P_{t\tau^*}d\tau^*=0. \tag{E9}$$

Pollution tax at home

Total differentiation of (147) yields

$$\Big[\frac{1}{n}p_YY_\tau(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)+\frac{1}{n^*}p^*{}_{Y^*}Y^*_\tau(X^*_{t^*}dt^*+X^*_\tau d\tau+X^*_{\tau^*}d\tau^*)$$
$$-(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)-(X^*_{t^*}dt^*+X^*_\tau d\tau+X^*_{\tau^*}d\tau^*)\Big]+a\Big[-p_\tau(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)$$
$$-q_\tau(Y_tdt+Y_\tau d\tau+Y_{\tau^*}d\tau^*)+\frac{1}{n}p_YY_\tau(X_tdt+X_\tau d\tau+X_{\tau^*}d\tau^*)$$
$$+\frac{1}{n^*}p^*{}_{Y^*}Y^*_\tau(X^*_{t^*}dt^*+X^*_\tau d\tau+X^*_{\tau^*}d\tau^*)+\widetilde{X_\tau}d\tau+Y_\tau dt\Big]=0. \tag{E9}$$

Collecting coefficients of $d\tau$, dt, dt^* and $d\tau^*$ we have

$$\begin{aligned} &\left[\left(\frac{1}{n}p_Y Y_\tau X_\tau + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_\tau - X_\tau - X^*_\tau\right) + a\left\{ -(p_\tau X_\tau + q_\tau Y_\tau)\right.\right. \\ &\left.\left. + \frac{1}{n}p_Y Y_\tau X_\tau + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_\tau + \widetilde{X_\tau}\right\}\right] d\tau + \left[\left(\frac{1}{n}p_Y Y_\tau X_t - X_t\right) + a\left\{ -(p_\tau X_t + q_\tau Y_t) + \frac{1}{n}p_Y Y_\tau X_t\right.\right. \\ &\left.\left. + Y_\tau\right\}\right] dt + \left[\left(\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{t^*} - X^*_{t^*}\right) + a\left\{\frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{t^*}\right\}\right] dt^* + \left[\left(\frac{1}{n}p_Y Y_\tau X_{\tau^*} + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{\tau^*}\right.\right. \\ &\left.\left. - X_{\tau^*} - X^*_{\tau^*}\right) + a\left\{ -(p_\tau X^*_\tau + q_\tau Y^*_\tau) + \frac{1}{n}p_Y Y_\tau X^*_\tau + \frac{1}{n^*}p^*_{Y^*}Y^*_\tau X^*_{\tau^*}\right\}\right] d\tau^* = 0; \end{aligned} \tag{E10}$$

which can be written as

$$W^P_{\tau\tau}d\tau + W^P_{\tau t}dt + W^P_{\tau t^*}dt^* + W^P_{\tau\tau^*}d\tau^* = 0. \tag{E11}$$