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Abstract:

This paper develops a theoretical model of directed lending policy, a specific kind of credit market intervention, in which regulators mandate lenders to supply a certain proportion of their loanable funds to certain underserved sectors or marginalized borrowers. In the absence of a directed lending policy, the optimal loan contract offered to the marginalized borrower is unfavourable compared to the non-marginalized borrower, even though both borrowers have an identical project. When directed lending policy is introduced, the optimal loan contract offered to marginalised borrower improves in the sense that the borrower receives higher loan at lower interest rate vis-à-vis the case without intervention. Further, we derive an expression for the optimal level of mandated lending that maximizes social welfare. Our results have two implications – first, directed lending policy is able to increase the flow of funds to the targeted sector and second, lenders can make positive profits even in the presence of such interventions as long as on the level of intervention is below a threshold.

Keywords: credit market inefficiency, directed lending program, intervention, mandated lending, priority sector

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I. Introduction

The importance of finance as a conduit of economic growth is aptly recognized. When internal sources of finance become inadequate and limited, external sources are sought out wherein financial intermediation plays a key role in determining who gets access to finance and to what extent. The banking system, one of the most important sources of funds, comes with the constraint of having to deal with imperfect information about the pool of borrowers. This problem of asymmetric information leads to 'credit rationing', a situation where among identical borrowers some borrowers get credit while some others are denied credit even if they are willing to pay the higher interest rate. Credit rationing may result in underinvestment in good opportunities because of unavailability or under-availability of credit to borrowers. While credit rationing can happen to any borrower, studies have found that poor people, persons living in the margins of the society and small firms suffer more from credit rationing (Avery, 1981; Japelli, 1990; Rangarajan, 2008; Brown et al., 2011; Nikaido et al., 2015). Thus, information asymmetry results in market failure requiring government intervention in the credit markets.

Few examples of credit market interventions are – Credit Guarantee Fund Scheme for micro and small enterprises in India, loans for small businesses backed by Small Business Administration agency in United States, Credit guarantee scheme through European Investment Fund in western Europe for small enterprises, Credit linked Subsidy Scheme under Pradhan Mantri Awas Yojana in India, Brazil's earmarked Credit policy, India's Priority sector lending policy and Philippines's Directed credit program. For the ease of analysis, these programs can be divided into three categories - credit subsidies, credit guarantee programs and directed lending policy. Under credit subsidy policy, government requires lenders to extend loan at a subsidised rate of interest. Under credit guarantee scheme, government acts as a guarantor for the loan and under directed lending policy, government mandates the banks to extend a certain amount of their loanable funds to some specific sectors.

The empirical investigation of the effects of government intervention programs in credit market has been of interest to many scholars. Many have argued that such intervention policies are not without costs as they distort the market-based outcomes and potentially lead to less optimal allocation of credit, particularly when credit directed towards certain sectors through intervention results in reduction of credit to other sectors with potentially higher returns (World Bank, 1989; Buttari, 1995). Further, many scholars have argued that even though intervention policies may improve credit flow to underserved sectors, increased flow of credit need not by itself translate into better economic outcomes (Schwarz, 1992; Vittas and Cho, 1995; Klapper and Zaidi, 2005). On the other hand, several empirical studies have concluded that credit market intervention policies are effective and beneficial for the target group. For example, loan guarantee schemes in US, UK, Central and Eastern Europe, France, Japan and Korea have been found to increase employment, investment, survival of new enterprises and supply of funds (Craig et al., 2008; Cowling and Siepel, 2013; Dvoulety et al. 2018; Lelarge et al., 2010; Uesugi et al., 2010; Kang and Heshmati, 2008). Directed lending programs have relaxed the credit constraints of small firms in India, machine tool producers in Japan and low/moderate income communities and minority borrowers in US (Banerjee and Duflo, 2014; Yadav and Sarma, 2021; Kale, 2016; Eastwood and Kohli, 1999; Calomiris and Himmelberg, 1993; Kruk and Haiduk, 2013; Barr, 2005). At the same time, Bhue et al. (2019) showed that Indian firms slow their pace of growth in order to get more credit under the scheme.

While there is a substantial empirical literature investigating whether or not credit market intervention policies are beneficial, only a few researchers have looked at it from a theoretical standpoint. The existing theoretical literature on this issue considers only a restricted set of intervention, viz., credit subsidies and credit guarantee schemes. In this paper, we build a theoretical model of credit market intervention by considering directed lending policy as the form of intervention. The primary objective of this paper is to add to sparse literature on theoretical modeling of government intervention programs in credit markets. More specifically, this paper will focus on developing a theoretical model of directed lending, a specific type of intervention, to address the imperfect outcome resulting from informational asymmetries between the borrowers and the lender.

In this paper, we attempt to model credit market imperfections based on some distinguishable characteristics borrowers, such as socio-economic strata or particular sector of the economy where they belong to. This is in line with how directed lending policies are formulated. For example, in the case of priority sector lending policy of India, credit supply is ensured to credit constrained sectors like micro and small enterprises, agricultural sector and underprivileged borrowers such as those from disadvantaged socio-economic groups and females. We will present a case of disadvantageous sector which could be redlined and later will see the implication of priority sector lending policy in this case.

The rest of the paper is organised as follows. Section 2 presents a review of the related literature. In section 3, we present our theoretical model of directed lending policy with some analysis of the results along with some simulation results. Thereafter, we attempt to find the optimal level of intervention in section 4. Section 5 concludes the paper.

II. Literature Review

The theory of credit rationing dwells on the premise that information asymmetry in credit market may manifest itself in various forms, viz., adverse selection, moral hazard and costly monitoring. Stiglitz and Weiss (1981) developed a theoretical model to explain this where they assume that lenders cannot observe the riskiness of different projects or the amount of effort the borrowers would put in the projects to make them successful. On the other hand, model of Williamson (1986) assumed that the borrowers may not have an incentive to truthfully reveal the outcome of the project. As a result, lenders would prefer to restrict credit flow instead of raising the interest rate as doing so might worsen the quality of lender's portfolio of loans. Such information asymmetry between the lender and the borrower can result in the phenomenon of credit rationing, wherein credit is denied by the lenders to some of the identical borrowers even when they are willing to pay higher interest rate. Credit rationing is an imperfect market equilibrium, in which the demand for loans exceeds supply but yet the lenders do not raise interest rate, instead credit is denied to some borrowers even though they are willing to pay a higher interest rate. In Stiglitz and Weiss (1981), credit rationing appears to be an equilibrium solution in the presence of information asymmetry. While the phenomenon of 'credit rationing' is explained by theoretical models, the theoretical literature is silent on how should the lender decide whom to lend and whom to 'ration out'. Empirical literature, however, has adequately documented that the 'rationed out' borrowers are largely poor and marginalized ones (Avery, 1981; Japelli, 1990; Rangarajan, 2008; Brown et al., 2011; Nikaido et al., 2015). Chaudhary and Jain (2017) in their natural experiment setting in Pakistan show that banks disproportionately reduced credit to new firms following increase in funding costs due to floods. The characteristics of firm owner also matter to advance credit to them like female managed firms are at disadvantage in Philippines and less financially developed countries of Europe, caste of firm owners in India, race and ethnicity in United States (Malapit, 2012; Muravyev et al., 2007; Rajesh and Sasidharan, 2018; Blanchflower et al., 2003; Cavalluzzo et al., 2002). These borrowers not only face difficulty in obtaining loans but also have to be contented with less loan and that too at a higher interest rate (Muravyev et al. 2007; Rajesh and Sasidharan, 2018; Blanchflower et al., 2003). Credit rationing of the poor and the marginalized has also been documented in the literature on 'redlining', which can be defined as a discriminatory practice against economically and racially disadvantaged individuals and households (Hillier, 2003; Ong and Stoll, 2007).

Credit market intervention policies in many countries aim at increasing credit supply to certain sectors of the economy or certain segments of the population that are credit constrained. We can classify such policies into three categories – interest rate subsidy (or credit subsidy), loan guarantee and directed lending policy. The empirical investigation of the effects of such intervention programs in the credit market has been of interest to many scholars. Many have argued that such intervention policies are not without costs as they distort the market-based outcomes and potentially lead to less optimal allocation of credit, particularly when credit directed towards certain sectors through intervention results in reduction of credit to other sectors with potentially higher returns (World Bank, 1989; Buttari, 1995). Further, it is also argued that even though intervention policies may improve credit flow to underserved sectors, increased flow of credit need not by itself translate into better economic outcomes (Schwarz, 1992; Vittas and Cho, 1995; Klapper and Zaidi, 2005). In a recent paper, Bhue et al. (2019) showed that Indian firms slow their pace of growth in order to get more credit under the priority sector lending scheme.

On the other hand, several empirical studies have concluded that credit market intervention policies are effective and beneficial for the target group. For example, loan guarantee schemes in US, UK, Central and Eastern Europe, France, Japan and Korea have been found to increase employment, investment, survival of new enterprises and supply of funds (Craig et al., 2008; Cowling and Siepel, 2013; Dvoulety et al. 2018; Lelarge et al., 2010; Uesugi et al., 2010; Kang and Heshmati, 2008). Directed lending programs have relaxed the credit constraints of small firms in India, machine tool producers in Japan and low/moderate income communities and minority borrowers in US (Banerjee and Duflo, 2014; Yadav and Sarma, 2021; Kale, 2016; Eastwood and Kohli, 1999; Calomiris and Himmelberg, 1993; Kruk and Haiduk, 2013; Barr, 2005). Thus, the empirical literature on efficacy of credit market intervention presents mixed evidence.

While empirical studies have paid some attention to evaluating government intervention programs in credit market, the same cannot be said about the theoretical literature. Only a few researchers have looked at this from a theoretical standpoint. The scant theoretical literature on credit market intervention is based on asymmetric information models. Further, the theoretical research on credit market interventions have looked at two programs viz. credit subsidies and loan guarantee schemes to reduce market inefficiencies, mainly adverse selection. Mankiw (1986) through his model of students' loan market, showed that mandating an interest rate at the risk-free level ensures that those students who have greater expected return obtain loan. In the model, intervention is effective to achieve greater social welfare than private outcome because the rate of interest is not dependent on the pool of applicants who apply for loan which itself is affected by the interest rate. Interest rate, by itself, cannot serve the purpose of screening indistinguishable borrowers. Hence, some theoretical models incorporate collateral requirement and/or probability of obtaining loan in their loan contract (Gale, 1989; Smith and Stutzer, 1989; Janda, 2011). Lenders, instead of offering one single contract to all the borrowers, offers different loan contracts for the borrowers to choose from in case of adverse selection. The loan contracts should be designed in a manner that the borrowers apply for loan (participation constraint) and choose the terms of the contract which were meant for them (incentive compatibility constraint). In this setting, the optimal contract between lender and borrower is determined by maximization of expected utility of the borrower subject to the constraint that lender receives an expected return that matches the risk-free return. The problem is in the solution itself, in the sense that the screening devices, collateral requirements or rationing

(Williamson, 1994; Rai, 2007) used by the private lenders to solve the problem of adverse selection reduces the overall welfare in terms of sum of expected utilities of the borrowers as defined by Gale (1989) or number of borrowers applying for loan defined by Latruffe and Fraser (2002). This happens because high risk borrowers put the low-risk borrowers at a disadvantage in the market. Disadvantage could be in the form of posting collateral (Gale, 1989) or fewer low risk borrowers getting loan (Gale, 1989). Hence, state intervention in such credit market could be beneficial, though not necessarily. Innes (1990) analyses the case of interest rate subsidy without giving any conclusive statement for government intervention.

The government intervention programs enter the theoretical models by a change in the constraints faced by the borrowers in their expected utility maximization exercise, either it affects the zero-profit condition of the lender (Gale, 1989; Smith and Stutzer, 1989; Williamson, 1994; Rai, 2007) or the incentive compatibility constraint or participation constraint of the borrower (Latruffe and Fraser, 2007). On the other hand, when the lender optimally chooses the rate of interest and rationing probabilities, then the expected return of the lender is modified by the introduction of loan guarantee or interest rate subsidy schemes (Janda, 2011). Another kind of intervention is when government directly supplies loans to those borrowers who are denied credit by private lenders. Williamson (1994) shows that when government gives direct loans to the borrowers by raising funds from private lenders then government simply displaces private loans leaving the impact of policy invariant. Rai (2007) demonstrates that if government is introduced as a co-financier with the private lender then government intervention is beneficial in reducing credit rationing and increasing the expected utility of the borrowers when the government is the first claimant to loan repayment.

The credit market inefficiencies suggest that there should be government intervention but any form of intervention program may not produce the desired results. If the pool of borrower is collateral constrained then loan guarantee schemes work and if the expected utility from the loan is below the reservation utility than interest rate subsidies work better as shown by simulation exercises done by Latruffe and Fraser (2007). Loan guarantee to high-risk borrowers rather than low-risk borrowers makes the loan contract offered to high-risk borrower more attractive leading to gain in efficiencies Gale (1989). The credit subsidies are favorable, from budgetary consideration, when there is a heterogeneity of project success whereas credit guarantees are preferred when there is a less chance of project success because subsidies are paid to all irrespective of the success or failure whereas guarantee is paid only to the failures (Janda, 2011).

In summary, the literature discusses the theoretical basis of imperfections in credit markets. Further, it is discussed that lenders use various screening devices like collateral requirement or restriction on grant of loan in order to reduce the effect of imperfections on their expected payoff. These instruments, by themselves, enhance the inefficiencies present in the market which presents the case for state intervention. The various credit intervention programs modify the optimization exercise of the borrower or lender as per the model and can lead welfare enhancing outcomes.

These models minimize the errors which formal sector like banks commit viz. giving loans to bad borrowers and denying loans to good borrowers, using credit intervention programs. These errors are committed for the simple reason that the banks have to not only differentiate between good and bad borrowers but also have to assess whether the projects are executed with best possible efforts and their outcomes are fairly verified. Hence, these models have addressed the issue of imperfect information which takes the form of adverse selection, moral hazard and costly state verification through government intervention programs.

However, there are another type of imperfections which are present in the financial markets where credit is rationed based on distinguishable characteristics, i.e., based on which sector or segment of the economy and society the borrower belongs to. As discussed earlier, there is a plenty of empirical literature that show rationing of credit for micro and small enterprises in the production sector or rationing of credit for backward castes, females etc. in consumption sector. Here, the information asymmetry problem arises due to lack of available credit histories, lack of book keeping records and so on on the part of the borrower, which itself could be due to the borrowers' exclusion from the formal financial system and due to the informal nature of his business. This particular problem is acute in developing counties where informal sector has a greater share of the market compared to developed countries. The opaque nature of these firms or households (unbankable entities) result in unfulfilled credit demand. Directed lending programs in different countries have been put in place to identify these cases and mandate bank lending. Hence, this paper will focus on this aspect of imperfect information where bank may deny credit to good borrower based on its distinguishable features. The theoretical models dealing with this issue are conspicuously absent in the literature which this paper attempts to address. We will model this problem and address it with directed lending program.

III. **The Model**

Suppose that there are two types of borrowers, indexed by Type 1 and type 2. Type 1 borrowers are opaque because they belong to marginalized and underprivileged sectors hence have no prior credit history while type 2 borrowers are from the privileged sectors with hard information on their creditworthiness.

The lender may grant loan to a borrower after monitoring information about the borrower. Information can take two forms, viz. hard information and soft information. Hard information can refer to collateral, wealth endowment or prior credit history whereas soft information can be borrower's entrepreneurial ability or trustworthiness. Type 1 or marginalized borrowers are the ones for whom hard information is difficult to ascertain because of reasons such as lack of book keeping, no prior access to bank credit and consequent lack of credit history etc. The information asymmetry here stems from the lack of credit history/lack of bank linkage due to the underprivileged nature of these firms or households, which leads to unfulfilled credit demand. These borrowers have private information about their creditworthiness which the lender is unable to observe. Hence the lender would either avoid lending to such category of borrowers, a phenomenon termed as "redlining" in the literature, or has to incur certain costs to ascertain the credit worthiness of the borrower before deciding to lend. So, we assume, lenders incur a screening cost, c per unit of loan amount to sanction loan to the type 1 borrowers whereas there is no screening cost for type 2 borrowers. The term type 1(type 2) borrower and marginalized (non-marginalized) borrower is used interchangeably in the paper. Further, we assume that there is no monitoring cost of the project for the lender.

The i^{th} borrower can undertake a risky project which yields Y_i in case of success and 0 in the case of failure (i=1,2). The probability of success of the project is given by δ. Thus, the two borrowers are identical in their risk profile, and have the identical probability of success. For simplicity, we assume that the output of the project depends only on one input i.e. funds/credit received from the lender. The return of the successful project follows production function given by $Y_i = L_i^{\alpha}$, where L_i is the amount of loan amount received by the i^th borrower and $0 < \alpha < 1$.

The role of the lender is to channelize funds from depositors to be lent to borrowers. The lender can obtain funds from the depositors at a unit cost of ρ. We assume that there is a single lender in the credit market similar to Janda (2011). The lender offers a loan contract comprising of loan amount and interest rate to the borrower. Hence, each loan contract is a pair (l_i, r_i) where l_i is the loaned amount and r_i is the interest rate. The loan contract is specific to the type of borrower. Hence, (l_1, r_1) is the loan contract offered to type 1 borrowers and (l_2, r_2) is the loan contract offered to type 2 borrowers. The lender receives the interest income in the case of successful project and loses the loaned amount in the case of failure. The lender chooses the loan contract that maximizes its expected profit given by:

$$\begin{aligned}\pi &= \delta[(1+r_1)l_1 - (1+\rho)l_1 - cl_1] + (1-\delta)[-(1+\rho)l_1 - cl_1] \\ &\quad + \delta[(1+r_2)l_2 - (1+\rho)l_2] + (1-\delta)[-(1+\rho)l_2] \\ &= \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1+l_2) - cl_1\end{aligned} \quad \ldots (1)$$

For simplicity, we will assume there is a single borrower in each category to represent it. A borrower gets net output, output after repayment of loan when the project succeeds and obtains nothing in the case of failure. Because, it is assumed that the borrowers are protected by limited liability clause. Hence, a representative borrower belonging to each type receives an expected utility of:

$$\begin{aligned}U_i &= \delta(Y_i - (1+r_i)l_i) + (1-\delta) * 0 \\ &= \delta(Y_i - (1+r_i)l_i)\end{aligned} \quad \ldots (2)$$

A. *Observable Private Information*

This is the ideal case of complete information where the lender can observe the private information of the borrower. Therefore, the lender will not incur any screening cost for type 1 borrower. The expected profit of the lender as mentioned in equation (1) is modified by exclusion of the term involving c. In this case, the optimal loan contract will be the one where the lender maximizes the expected return from the project subject to the participation constraints of the borrowers:

PROBLEM 0 :

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1 + l_2)$$

$$subject\ to$$

$$\delta(Y_1 - (1+r_1)l_1) \geq 0$$

$$\delta(Y_2 - (1+r_2)l_2) \geq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

The solution of this problem[2] is given by:

SOLUTION 0 :

$$r_1^* = r_2^* = \frac{1+\rho}{\delta\alpha} - 1$$

$$l_1^* = l_2^* = \left[\frac{\delta\alpha}{1+\rho}\right]^{\frac{1}{1-\alpha}}$$

The expression for interest rate and loan amounts are intuitive. Interest rate is an increasing function of cost of borrowing and decreasing function of success probabilities as well as elasticity of output in the borrower's production function. Additionally, the interest rate is a decreasing and convex function of amount of loan offered. Hence, the increase in cost of borrowing decreases the loan sanctioned substantially. At the same time, the increase in success probability increases the loan amount.

The solution shows that the lender offers identical loan contract to both the type of borrowers (i.e., loan amount offered and interest rate is identical). This is the case where the lender advances loan based on the riskiness of the project rather than that of the borrowers. Since the borrowers have same risky project to invest in, the lender should offer the identical loan contract. Here, we present a case of no discrimination between borrowers which requires no policy intervention in the credit market. This equilibrium is also socially efficient because the project is financed only where the expected return from the project (δY_i) is greater than the cost of funds (ρL_i).

[2] Proof of the solution in Appendix A

Proposition 1: *In the absence of information asymmetry, both borrowers receive the same loan contract and loanable funds is divided between two equally*

The rest of the paper will focus on in what manner terms of loan contract changes with the presence of unobservable private information of the borrower to the lender. The effect of directed lending policy will be examined in this context. In case the directed lending policy turns out to be effective, the paper will make an attempt to find optimal level of intervention.

B. *Unobservable Private Information and No Government Intervention*

Let us now consider the case in which lender cannot observe the soft information of the borrower when hard information is either unavailable or limited. This is true for type 1 borrower only. For that reason, the lender has to incur costs denoted by *c* per unit of loan amount. The lender optimizes the expected payoff given in equation (1) subject to the participation constraints of each type of borrower given in equation (2). And so, the optimization exercise becomes:

PROBLEM 1 :

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1 + r_1)l_1 + (1 + r_2)l_2] - (1 + \rho)(l_1 + l_2) - cl_1$$

$$\textit{subject to}$$

$$\delta(Y_1 - (1 + r_1)l_1) \geq 0$$

$$\delta(Y_2 - (1 + r_2)l_2) \geq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

The solution[3] of the associated problem is given by:

SOLUTION 1 :

$$r_1^* = \frac{1 + \rho + c}{\delta\alpha} - 1 \qquad\qquad r_2^* = \frac{1 + \rho}{\delta\alpha} - 1$$

[3] Proof given in Appendix B

$$l_1^* = \left[\frac{\delta\alpha}{1+\rho+c}\right]^{\frac{1}{1-\alpha}} \qquad l_2^* = \left[\frac{\delta\alpha}{1+\rho}\right]^{\frac{1}{1-\alpha}}$$

The solution shows that the lender offers two different loan contracts to the two types of borrowers for the same risky project. Although the loan contract of the type 2 borrower remains the same as the previous case, the loan contract offered to type 1 borrower is less favorable. Type 1 borrower is offered lower loan amount, that too at a higher interest rate. The premium on interest rate can be justified on the ground of covering up the extra screening costs incurred, but the burden falls solely on type 1 borrower leaving the type 2 borrower unaffected. The increased interest rate reduces the loan disbursement to type 1 borrowers. Thus, making the loan contract for type 1 borrower twice as unfavorable. In the extreme case where screening costs increase infinitely, these borrowers are red-lined by the lender. Our findings are in line with the findings of empirical literature discussed in the previous section that these marginalized borrowers are treated differently in the loan market (Muravyev et al. 2007; Rajesh and Sasidharan, 2018; Blanchflower et al., 2003). Thus, the model presents a case for government intervention in credit market to mitigate these inefficiencies. One such policy that is directed towards these sectors or segments of the market is directed lending policy discussed in the next subsection.

Proposition 2: *In the absence of intervention, the marginalised borrowers receive an unfavorable contract vis-à-vis the non-marginalised borrowers.*

C. *Unobservable Private Information and Government Intervention*

Directed lending program mandates that the lender offers a minimum amount of their total loanable funds to the informationally opaque borrowers. Hence, the intervention is introduced as an additional constraint in the optimization problem faced by the lender. The constraint is given by:

$$\frac{l_1}{l_1 + l_2} \geq \beta \qquad \ldots (3)$$

where β is the minimum proportion of total loanable funds to be offered to type 1 borrowers and $0 \leq \beta \leq 1$. The modified maximization problem of the lender under this intervention is as follows:

PROBLEM 2 :

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1 + r_1)l_1 + (1 + r_2)l_2] - (1 + \rho)(l_1 + l_2) - cl_1$$

$$\textit{subject to}$$

$$\delta(Y_1 - (1 + r_1)l_1) \geq 0$$

$$\delta(Y_2 - (1 + r_2)l_2) \geq 0$$

$$\frac{l_1}{l_1 + l_2} \geq \beta$$

$$r_1, r_2, l_1, l_2 \geq 0$$

The solution[4] to this problem is as follows:

SOLUTION 2 :

$$r_1^* = \left(\frac{1 + \rho + \beta c}{\delta\alpha}\right)\left(\frac{\beta^{\alpha-1}}{(1 - \beta)^{\alpha} + \beta^{\alpha}}\right) - 1 \qquad r_2^* = \left(\frac{1 + \rho + \beta c}{\delta\alpha}\right)\left(\frac{(1 - \beta)^{\alpha-1}}{(1 - \beta)^{\alpha} + \beta^{\alpha}}\right) - 1$$

$$l_1^* = \left[\left(\frac{\delta\alpha}{1 + \rho + \beta c}\right)\left(\frac{(1 - \beta)^{\alpha} + \beta^{\alpha}}{\beta^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}} \qquad l_2^* = \left[\left(\frac{\delta\alpha}{1 + \rho + \beta c}\right)\left(\frac{(1 - \beta)^{\alpha} + \beta^{\alpha}}{(1 - \beta)^{\alpha-1}}\right)\right]^{\frac{1}{1-\alpha}}$$

As is in the solution, the screening costs is shared by both the types of borrowers as the term appears in both the loan contracts. Even though type 2 borrower shares some screening costs incurred for type 1 borrowers, they are the ones who still get more loan at lower interest rate than type 1 borrowers. In either case of intervention, we observed that non-marginalized borrowers are better off than marginalized ones.

Proposition 3: *The non-marginalised borrowers will always have a better terms of contract vis-à-vis marginalised borrowers, regardless of the intervention policy.*

Let's turn to the terms of loan contract marginalized borrowers for whom the directed lending policy was designed. There is no clear answer whether the terms improve or worsen for any

[4] Proof given in Appendix C.

combination of parameters i.e., intervention level, screening costs, output elasticities and opportunity cost of raising funds. But if the parameters are restricted then terms of loan for the marginalized borrowers will certainly improve. If the level of intervention is greater than 50% then type 1 borrowers are able to obtain better loan terms. Even if the intervention level is less that 50%, there is some intermediate levels wherein type 1 will be better off. Therefore, credit market intervention in the form of directed lending can benefit the targeted sectors or segments of the economy.

<u>Proposition 4</u>: *Directed lending policy improve the terms of contract for marginalised borrowers under certain parameter configuration*

3.1 Regardless of different parameter values, the terms of contract for marginalised borrowers will improve if the level of intervention is greater than 50%

3.2 In case intervention is less than 50%, then the terms of contract will improve if

$$\frac{1+\rho+\beta c}{1+\rho+c} \leq \beta\left(1+\left(\frac{1-\beta}{\beta}\right)^{\alpha}\right)$$

D. *<u>Financial viability</u>*

The previous section has established the efficacy of the directed lending policy for the targeted sector/segment; however, the financial viability of the lender is an important consideration. In order to assess the impact of this program on lender's expected profit, we compute the optimized expected profit of the lender, which is as follows[5]:

$$\Pi = \delta^{\frac{1}{1-\alpha}}\,\alpha^{\frac{\alpha}{1-\alpha}}\,(1-\alpha)(1+\rho+\beta c)^{\frac{-\alpha}{1-\alpha}}\,[(1-\beta)^{\alpha}+\beta^{\alpha}]^{\frac{1}{1-\alpha}} \qquad \ldots (4)$$

It can be seen that the expected profit of the lender is always positive regardless of the value of the intervention parameter β, keeping other parameters ceteris paribus. . The possible reason behind this result is that the projects that are socially efficient[6], if financed, will result in positive payoff for the lender. In our model, we have assumed that the borrowers have identical projects with identical success probabilities so the lender should offer identical loan contract in order to

[5] Insert the solution of equilibrium loan amounts and interest rates obtained in Problem 2 in equation 1

[6] The equilibrium is socially efficient when the expected return from the project (δY_i) is greater than the cost of funds (ρL_i) as mentioned in Problem 0.

maximize its profits. But the lender does not offer the same terms of loan to the two types of borrowers in the presence of information opacity in type 1 borrower. The directed lending policy not only improves the situation for the marginalized borrowers, but the policy also does not lead to generate negative profits to the lender because even the marginalized borrowers are good borrowers. However, the expected profit of the lender falls with an increase in screening costs for the type 1 borrower. This happens because the screening cost for the marginalized borrower is a social waste which reduces the efficiency of the outcome, just like Gale (1989) who considered collateral requirement as a waste.

Proposition 5: *In the presence of directed lending policy, banks are able to earn positive profits.*

IV. Welfare Analysis

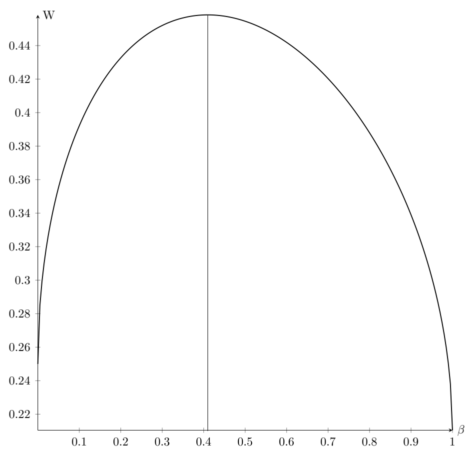
This section will focus on analyzing the welfare aspect of the government intervention program of directed lending policy. We measure total welfare as sum total of lender's expected profit and the total output produced in the economy. With this definition in mind, we will investigate the change in welfare due to intervention in the credit market and find an optimal intervention level, if exists.

A. Welfare function under intervention

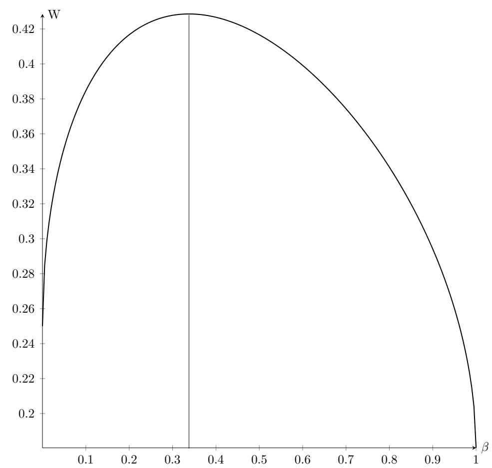
With directed lending policy, the total welfare (W) as a function of the intervention parameter β is as follows:

$$W(\beta) = (\delta\alpha)^{\frac{\alpha}{1-\alpha}} (1 + \delta - \alpha\delta)(1 + \rho + \beta c)^{\frac{\alpha}{\alpha-1}} [(1-\beta)^{\alpha} + \beta^{\alpha}]^{\frac{1}{1-\alpha}} \quad \dots (5)$$

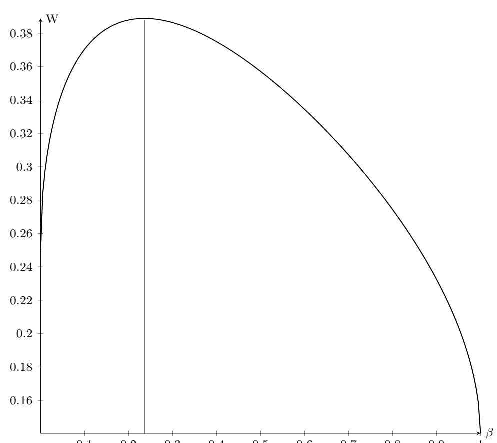
Figure 1 presents the graph of total welfare function as a function of β for different parametric specifications. In each of the four figures, output elasticity α is taken to be 0.5, success probability δ of the project is ½ and opportunity cost of raising fund ρ is 0.25. The screening cost increases as we move from part(a) to part (d) of the figure. In part (a), the screening cost is equal to cost of raising funds i.e., ¼ which increases to ½ in part (b), further increasing to 1 in part (c) and finally to 2 per unit of loan amount in part (d). In all the figures, we note that total welfare is non-negative for any level of intervention in the credit market. Furthermore, it is important to note that the welfare function is concave in β. The concave nature of the function implies that it is possible to find an intervention level that can maximize the total welfare function. There are a few noteworthy points in Figure 1. As we move from part (a) to part (d) which shows an increase in screening cost while keeping other parameters at a fixed level, the level of intervention that maximizes the welfare decreases and there is also a decrease in maximal value of the welfare. Hence, screening cost plays an important role in welfare effects of intervention.



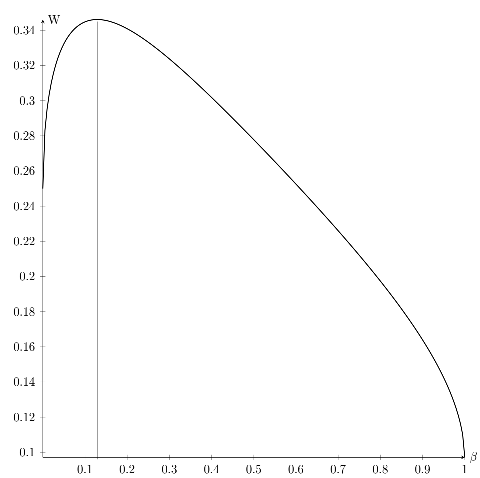
(a) $\alpha = \frac{1}{2}, \rho = \frac{1}{4}, \ \delta = \frac{1}{2}, c = \frac{1}{4}$



(b) $\alpha = \frac{1}{2}, \rho = \frac{1}{4}, \ \delta = \frac{1}{2}, c = \frac{1}{2}$



(c) $\alpha = \frac{1}{2}, \rho = \frac{1}{4}, \ \delta = \frac{1}{2}, c = 1$



(d) $\alpha = \frac{1}{2}, \rho = \frac{1}{4}, \ \delta = \frac{1}{2}, c = 2$

FIGURE 1: Welfare in the case of directed lending policy as a function of β for different parameter combinations.

B. *Welfare comparisons*

Although the efficacy of the directed lending policy for marginalized borrowers has been established in Section III, the welfare effects of the policy are still unclear. Therefore, we will compare total welfare with intervention in credit markets with total welfare without intervention. Welfare with intervention policy is given in Equation (5). Welfare without intervention is as follows:

$$W_1 = (\delta\alpha)^{\frac{\alpha}{1-\alpha}}(1+\delta-\alpha\delta)[(1+\rho+c)^{\frac{\alpha}{\alpha-1}}+(1+\rho)^{\frac{\alpha}{\alpha-1}}]$$

If we compare these two equations, there is no unambiguous result to tell under which circumstance the welfare is greater. The result crucially depends on three parameters: opportunity cost of loan (ρ), screening cost (c) and output elasticity (α). Thus, we cannot say that the policy of directed lending is always welfare enhancing. Nonetheless, there is a parameter configuration for the production function and costs which is welfare enhancing in the presence of directed lending policy.

Proposition 6: *Total welfare increases with intervention for some level of intervention i.e.,*

$$\frac{\left(\frac{1+\rho+c}{1+\rho}\right)^{\frac{\alpha}{\alpha-1}}+1}{\left(\frac{1+\rho+\beta c}{1+\rho}\right)^{\frac{\alpha}{\alpha-1}}} \leq [(1-\beta)^{\alpha}+\beta^{\alpha}]^{\frac{1}{1-\alpha}}$$

C. *Optimal level of Intervention*

From the welfare standpoint, the objective of this section is to find an optimal level of intervention through directed lending policy in the credit markets. In previous section, we have shown that the welfare function under intervention is concave, so it possible to find a unique level of intervention for which total welfare is maximized. Additionally, the expected profit of the lender (equation (4)) is proportional to the total welfare of the economy (equation (5)); so, the level of β which maximizes total welfare will also maximize profit of the lender.

When the total welfare function given in equation (5) is maximized with respect to β, the optimal level of intervention is given by[7]:

$$\beta^* = \frac{1}{1 + \left(\frac{1+\rho+c}{1+\rho}\right)^{\frac{1}{1-\alpha}}} \quad \dots (6)$$

The sum of opportunity cost and screening cost $(1 + \rho + c)$ is defined as the cost of fund to the marginalized borrowers, while opportunity cost relative $(1 + \rho)$ is defined as the cost of funds to the non-marginalized borrowers. The ratio of these costs to the bank plays an important role in the optimal intervention level at which economy benefits. At one extreme of zero screening cost, the two costs are equal which gives us Problem 0 of no distinction, and β^* should be ½. At the other extreme case of infinite screening cost, β^* tends to zero. If there is any positive screening cost then the ratio will always be greater than 1. Greater the degree of opaqueness for the marginalized borrowers, greater will be the screening costs. With an increase in screening costs, the ratio of two costs increases and β^* will decrease.

Figure 2 traces the level of intervention as a function of relative cost, given different levels of output elasticities. Each curve in fig.2 corresponds to one value of α, such that $\alpha \in (0,1)$ and plots β^* in equation (6) as the ratio of relative costs increases. It can be seen that β^*=½ when relative costs are equal and approaches zero as the relative costs increases indefinitely. The figure also shows that β^* falls as the relative cost increases, for any level of production parameter. At the same time, the fall in β^* is faster as output elasticity increases from ¼ to ¾.

[7] Proof in Appendix D

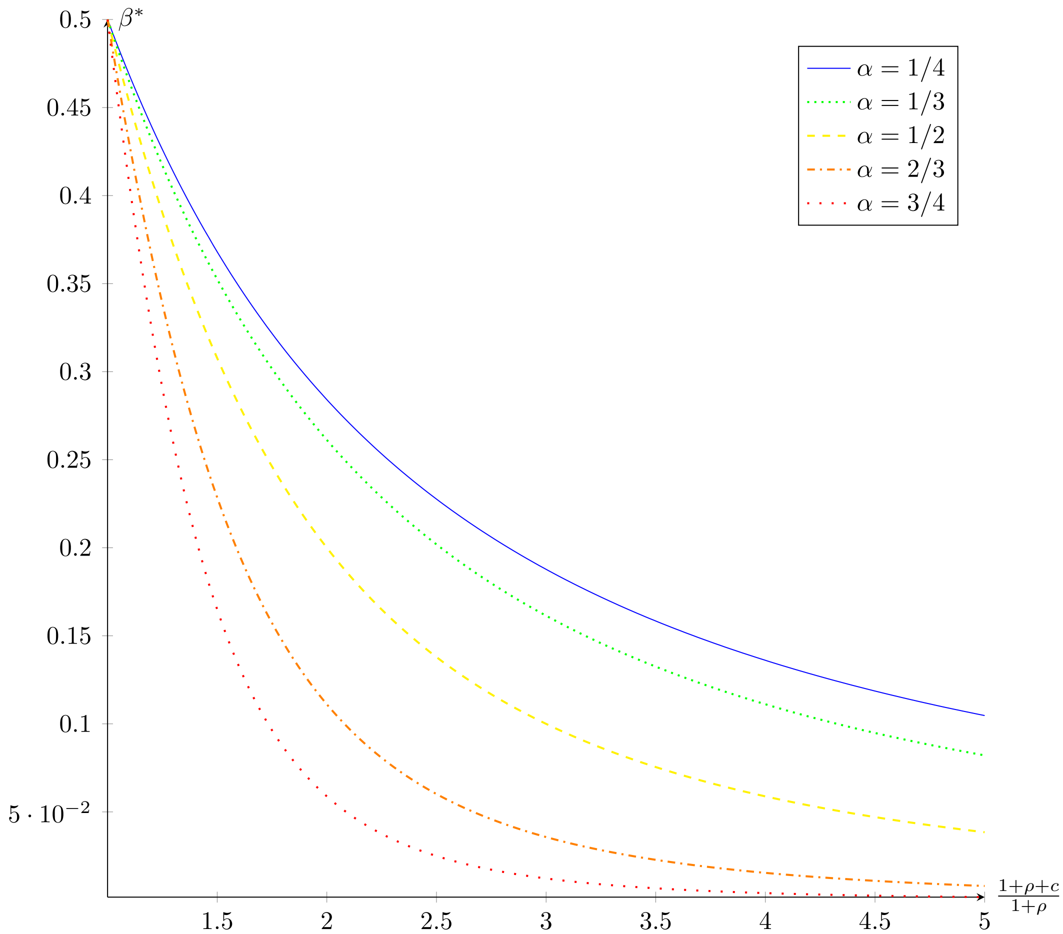
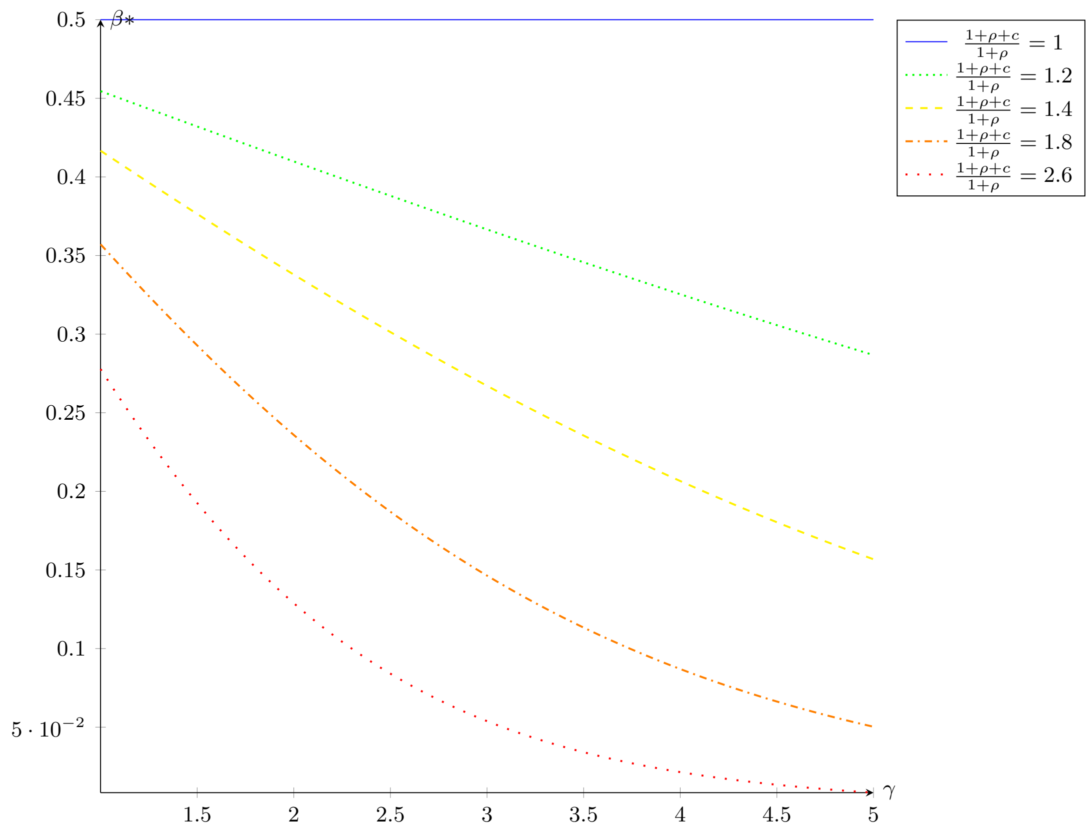


FIGURE 2: Relation between optimal value of intervention and relative cost of marginal borrower vis-à-vis non-marginal borrower

Besides relative costs, output elasticity (α) also features in equation (6) of β^*. The plot of intervention level as a function of production parameter for different relative costs is given in Figure 3. The figure considers five levels of relative costs, starting from 1 and increasing up to 2.6. The production parameter considered is inverse of one minus output elasticity ($\gamma = \frac{1}{1-\alpha}$). Since α lies between zero and one, γ is greater than 1. So, a single curve in the figure draws β^* as the value of γ increases. It is observed from the figure 3 that intervention should be go away as output elasticity is closer to one. If the output elasticity is closer to zero then β^* can go up to ½. Therefore, we have shown that there exists a case in point of higher level of intervention in credit markets through directed lending program for a lower relative costs and lower values of α .



<u>FIGURE 3</u>: Relation between optimal value of intervention and production parameter ($\gamma = \frac{1}{1-\alpha}$)

V. <u>Conclusion</u>

This paper develops a theoretical model of credit market intervention program of directed lending to address the problem of credit market inefficiencies. We believe that the gap in the literature on theoretical models of government intervention program exists because models attempted in the existing literature mostly focus on modeling for inefficiencies created by adverse selection and moral hazard while overlooking other inefficiencies. Whereas, our model captures the inefficiency in credit market that exist for the marginalized sectors or segment of the economy for whom directed lending program is framed. The theoretical model developed in Section III shows the effectiveness of the directed lending program in improving the social outcome. Further, the solution in Section IV indicates that the mandatory lending to the marginal borrowers should not exceed half of the total loanable funds. The mandated lending should be maximum when relative cost of marginalized borrower to non-marginalized borrower is the least.

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Appendix A:

Kuhn-Tucker Formulation for Problem 0 is as follows:

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1 + l_2)$$

$$\textit{subject to}$$

$$-\delta(Y_1 - (1+r_1)l_1) \leq 0$$

$$-\delta(Y_2 - (1+r_2)l_2) \leq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

Kuhn-Tucker Lagrangian:

$$\mathcal{L} = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1 + l_2) - \lambda_1\big(-\delta(Y_1 - (1+r_1)l_1)\big) - \lambda_2\big(-\delta(Y_2 - (1+r_2)l_2)\big)$$

First Order Conditions:

$$\frac{\partial \mathcal{L}}{\partial r_1} = \delta l_1 - \lambda_1 \delta l_1 = \delta(1-\lambda_1)l_1 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$\frac{\partial \mathcal{L}}{\partial r_2} = \delta l_2 - \lambda_2 \delta l_2 = \delta(1-\lambda_2)l_2 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

$$\frac{\partial \mathcal{L}}{\partial l_1} = \delta(1+r_1) - (1+\rho) + \lambda_1\delta\big(\alpha l_1^{\alpha-1} - (1+r_1)\big) \leq 0 \ldots\ldots\ldots (iii)$$

$$\frac{\partial \mathcal{L}}{\partial l_2} = \delta(1+r_2) - (1+\rho) + \lambda_2\delta\big(\alpha l_2^{\alpha-1} - (1+r_2)\big) \leq 0 \ldots\ldots\ldots (iv)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \delta(Y_1 - (1+r_1)l_1) \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots (v)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \delta(Y_2 - (1+r_2)l_2) \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots (vi)$$

With Complementary Slackness conditions:

(vii) $r_1 \frac{\partial \mathcal{L}}{\partial r_1} = 0$, $(viii)$ $r_2 \frac{\partial \mathcal{L}}{\partial r_2} = 0$, (ix) $l_1 \frac{\partial \mathcal{L}}{\partial l_1} = 0$, (x) $l_2 \frac{\partial \mathcal{L}}{\partial l_2} = 0$, (xi) $\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0$, (xii) $\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = 0$

Combining equations (i) and (vii), we get $\delta(1-\lambda_1)l_1 r_1 = 0$. We assume strictly positive success probability which gives $\lambda_1 = 1$ if loan amount and interest rate is zero. Similarly, we get $\lambda_2 = 1$ using equations (ii) and (viii). Using the production function, equation (v) and (vi) becomes $l_1^{\alpha-1} = 1 + r_1$ and $l_2^{\alpha-1} = 1 + r_2$. Inserting the value of λ_1 and equation (v) in equation (iii) will result in expression for r_1^0. Similarly, insert the value of λ_2 and equation (vi) in equation (iv) will result in expression for r_2^0. To obtain l_1^0 *and* l_2^0, substitute r_1^0 *and* r_2^0 in equations (v) and (vi) respectively.

Second Order Conditions:

The constrained optimization problem can be converted into an unconstrained optimization exercise by inserting the constraints in the objective function. In this case, we could do so because the constraints are binding. Following this procedure, the unconstrained problem is:

$$\pi = \delta(l_1^\alpha + l_2^\alpha) - (1+\rho)(l_1 + l_2)$$

$$\frac{\partial \pi}{\partial l_1} = \delta\alpha l_1^{\alpha-1} - (1+\rho)$$

$$\frac{\partial \pi}{\partial l_2} = \delta\alpha l_2^{\alpha-1} - (1+\rho)$$

$$\frac{\partial^2 \pi}{\partial l_1^2} = \delta\alpha(\alpha-1)\, l_1^{\alpha-2} < 0$$

$$\frac{\partial^2 \pi}{\partial l_2^2} = \delta\alpha(\alpha-1)\, l_2^{\alpha-2} < 0$$

$$\frac{\partial^2 \pi}{\partial l_1 l_2} = 0$$

Appendix B:

Kuhn-Tucker Formulation for Problem 1 is as follows:

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1 + l_2) - cl_1$$

$$subject\ to$$

$$-\delta(Y_1 - (1+r_1)l_1) \leq 0$$

$$-\delta(Y_2 - (1+r_2)l_2) \leq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

Kuhn-Tucker Lagrangian:

$$\mathcal{L} = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1 + l_2) - cl_1 - \lambda_1\big(-\delta(Y_1 - (1+r_1)l_1)\big) - \lambda_2\big(-\delta(Y_2 - (1+r_2)l_2)\big)$$

First Order Conditions:

$$\frac{\partial \mathcal{L}}{\partial r_1} = \delta l_1 - \lambda_1 \delta l_1 = \delta(1-\lambda_1)l_1 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$\frac{\partial \mathcal{L}}{\partial r_2} = \delta l_2 - \lambda_2 \delta l_2 = \delta(1-\lambda_2) l_2 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

$$\frac{\partial \mathcal{L}}{\partial l_1} = \delta(1+r_1) - c - (1+\rho) + \lambda_1 \delta\left(\alpha l_1^{\alpha-1} - (1+r_1)\right) \leq 0 \ldots . (iii)$$

$$\frac{\partial \mathcal{L}}{\partial l_2} = \delta(1+r_2) - (1+\rho) + \lambda_2 \delta\left(\alpha l_2^{\alpha-1} - (1+r_2)\right) \leq 0 \ldots\ldots\ldots (iv)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \delta(Y_1 - (1+r_1) l_1) \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (v)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \delta(Y_2 - (1+r_2) l_2) \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (vi)$$

With Complementary Slackness conditions:

(vii) $r_1 \frac{\partial \mathcal{L}}{\partial r_1} = 0$, $(viii)$ $r_2 \frac{\partial \mathcal{L}}{\partial r_2} = 0$, (ix) $l_1 \frac{\partial \mathcal{L}}{\partial l_1} = 0$, (x) $l_2 \frac{\partial \mathcal{L}}{\partial l_2} = 0$, (xi) $\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0$, (xii) $\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = 0$

Combining equations (i) and (vii), we get $\delta(1-\lambda_1) l_1 r_1 = 0$. We assume strictly positive success probability which gives $\lambda_1 = 1$ if loan amount and interest rate is zero. Similarly, we get $\lambda_2 = 1$ using equations (ii) and (viii). Using the production function, equation (v) and (vi) becomes $l_1^{\alpha-1} = 1 + r_1$ and $l_2^{\alpha-1} = 1 + r_2$. Inserting the value of λ_1 and equation (v) in equation (iii) will result in expression for r_1^1. Similarly, insert the value of λ_2 and equation (vi) in equation (iv) will result in expression for r_2^1. To obtain l_1^1 *and* l_2^1, substitute r_1^1 *and* r_2^1 in equations (v) and (vi) respectively.

Second Order Conditions:

The constrained optimization problem can be converted into an unconstrained optimization exercise by inserting the constraints in the objective function. In this case, we could do so because the constraints are binding. Following this procedure, the unconstrained problem is:

$$\pi = \delta(l_1^\alpha + l_2^\alpha) - (1+\rho)(l_1 + l_2) - c l_1$$

$$\frac{\partial \pi}{\partial l_1} = \delta \alpha l_1^{\alpha-1} - (1 + \rho + c)$$

$$\frac{\partial \pi}{\partial l_2} = \delta \alpha l_2^{\alpha-1} - (1 + \rho)$$

$$\frac{\partial^2 \pi}{\partial l_1^2} = \delta \alpha (\alpha - 1)\, l_1^{\alpha-2} < 0$$

$$\frac{\partial^2\pi}{\partial l_2^2} = \delta\alpha(\alpha-1)\, l_2^{\alpha-2} < 0$$

$$\frac{\partial^2\pi}{\partial l_1 l_2} = 0$$

Appendix C:

Kuhn-Tucker Formulation for Problem 2 is as follows:

$$\max\ \pi(r_1, r_2, l_1, l_2) = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1+l_2) - cl_1$$

$$subject\ to$$

$$-\,\delta(Y_1 - (1+r_1)l_1) \leq 0$$

$$-\delta(Y_2 - (1+r_2)l_2) \leq 0$$

$$\frac{\beta}{1-\beta} - \frac{l_1}{l_2} \leq 0$$

$$r_1, r_2, l_1, l_2 \geq 0$$

Kuhn-Tucker Lagrangian:

$$\mathcal{L} = \delta[(1+r_1)l_1 + (1+r_2)l_2] - (1+\rho)(l_1+l_2) - cl_1 - \lambda_1\big(-\delta(Y_1 - (1+r_1)l_1)\big) - \lambda_2\big(-\delta(Y_2 - (1+r_2)l_2)\big) - \mu\left(\frac{\beta}{1-\beta} - \frac{l_1}{l_2}\right)$$

First Order Conditions:

$$\frac{\partial\mathcal{L}}{\partial r_1} = \delta l_1 - \lambda_1\delta l_1 = \delta(1-\lambda_1)l_1 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$\frac{\partial\mathcal{L}}{\partial r_2} = \delta l_2 - \lambda_2\delta l_2 = \delta(1-\lambda_2)l_2 \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

$$\frac{\partial\mathcal{L}}{\partial l_1} = \delta(1+r_1) - c - (1+\rho) + \lambda_1\delta\big(\alpha l_1^{\alpha-1} - (1+r_1)\big) + \frac{\mu}{l_2} \leq 0 \ldots.(iii)$$

$$\frac{\partial\mathcal{L}}{\partial l_2} = \delta(1+r_2) - (1+\rho) + \lambda_2\delta\big(\alpha l_2^{\alpha-1} - (1+r_2)\big) - \frac{\mu l_1}{l_2^2} \leq 0 \ldots\ldots(iv)$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_1} = \delta(Y_1 - (1+r_1)l_1) \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(v)$$

$$\frac{\partial\mathcal{L}}{\partial\lambda_2} = \delta(Y_2 - (1+r_2)l_2) \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(vi)$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \frac{\beta}{1-\beta} - \frac{l_1}{l_2} \leq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (vii)$$

With Complementary Slackness conditions:

$(viii)$ $r_1 \frac{\partial \mathcal{L}}{\partial r_1} = 0$, (ix) $r_2 \frac{\partial \mathcal{L}}{\partial r_2} = 0$, (x) $l_1 \frac{\partial \mathcal{L}}{\partial l_1} = 0$, (xi) $l_2 \frac{\partial \mathcal{L}}{\partial l_2} = 0$, (xii) $\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0$, (xiii) $\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = 0$, (xiv) $\mu \frac{\partial \mathcal{L}}{\partial \mu} = 0$

Combining equations (i) and (viii), we get $\delta(1-\lambda_1)l_1 r_1 = 0$. We assume strictly positive success probability which gives $\lambda_1 = 1$ if loan amount and interest rate is zero. Similarly, we get $\lambda_2 = 1$ using equations (ii) and (ix). Using the production function, equation (v) and (vi) becomes $l_1^{\alpha-1} = 1 + r_1$ and $l_2^{\alpha-1} = 1 + r_2$. Insert the value of λ_1 *and* λ_2 and equations (v) and (vi) and solve equations (iii), (iv) and (viii) simultaneously to obtain r_1^2 *and* r_2^2. To obtain l_1^2 *and* l_2^2, substitute r_1^2 *and* r_2^2 in equations (v) and (vi) respectively.

Second Order Conditions:

The constrained optimization problem can be converted into an unconstrained optimization exercise by inserting the constraints in the objective function. In this case, we could do so because the constraints are binding. Following this procedure, the unconstrained problem is:

$$\pi = \delta l_2^{\alpha}\left(1 + \left(\frac{\beta}{1-\beta}\right)^{\alpha}\right) - l_2\left(\frac{1+\rho+\beta c}{1-\beta}\right)$$

$$\frac{\partial \pi}{\partial l_2} = \delta \alpha l_2^{\alpha-1}\left(1 + \left(\frac{\beta}{1-\beta}\right)^{\alpha}\right) - \left(\frac{1+\rho+\beta c}{1-\beta}\right)$$

$$\frac{\partial^2 \pi}{\partial l_2^2} = \delta \alpha(\alpha-1) l_2^{\alpha-2}\left(1 + \left(\frac{\beta}{1-\beta}\right)^{\alpha}\right) < 0$$

Appendix D:

$$max\, W(\beta) = (\delta\alpha)^{\frac{\alpha}{1-\alpha}}\,(1+\delta-\alpha\delta)(1+\rho+\beta c)^{\frac{\alpha}{\alpha-1}}\,[(1-\beta)^{\alpha}+\beta^{\alpha}]^{\frac{1}{1-\alpha}}$$

Using Envelope Theorem:

$$\frac{dW}{d\beta} = W\left(\frac{\alpha}{1-\alpha}\right)\left[\frac{\beta^{\alpha-1}-(1-\beta)^{\alpha-1}}{\beta^{\alpha}+(1-\beta)^{\alpha}} - \frac{c}{1+\rho+\beta c}\right]$$

Setting the above equation equal to zero, we can solve for β^*.

$$\frac{d^2W}{d\beta^2} = \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{\beta^{\alpha-1} - (1-\beta)^{\alpha-1}}{\beta^{\alpha} + (1-\beta)^{\alpha}} - \frac{c}{1+\rho+\beta c}\right]\frac{dW}{d\beta}$$

$$+ \left(\frac{\alpha}{1-\alpha}\right) W \left[(\alpha-1)\left(\frac{\beta^{\alpha-2} + (1-\beta)^{\alpha-2}}{\beta^{\alpha} + (1-\beta)^{\alpha}}\right) - \alpha + \frac{c^2}{(1+\rho+\beta c)^2}\right]$$

Evaluating at β^*,

$$\frac{d^2W}{d\beta^2} = -W\left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(1-\alpha)}{\beta(1-\beta)}\left(\frac{1+\rho+(1-\beta)c}{1+\rho+\beta c}\right) + \alpha - \frac{c^2}{(1+\rho+\beta c)^2}\right] < 0$$